The Arching of Beams
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Abstract:

The following solution was found using meditative techniques. The problem of arching of beams for dead load and live load is presented for infinite beams under an elastic media sitting on columns. The problem starts by solving the biharmonic equation using Fourier series. The solution presented is for a uniform load and gravity load. Other potential solution is architected at the end of the article. Charts and tables for simple span beam and fixed span beam is presented for an infinite elastic media and a finite elastic media with a given height. This problem has never been solved before.

Introduction:

The ancient problem of curved beams arching, explored by the Greeks and Romans, is applied, in this paper, to a straight beam instead of an arch. The following solution offers a new insight into the phenomenon of a straight beam behaving as an arch. Using finite element (FE), one observes the arching action when looking at the load distribution of a stress diagram. How, though, can FE be used to find the reductions in moment and shear due to the redistribution of the dead and live loads caused by arching? This dilemma is especially apparent in the analysis of concrete because FE does not accept reinforcing rebar; instead, 3-D analysis is needed. Additionally, modeling becomes very cumbersome if FE analysis must be performed for every beam in a structure. It would be much more productive if one knew the arching factors beforehand. Elasticity will allow the exploration of this phenomenon in all materials, including soils (for small strains).

In a previous paper, the problem was solved with an infinite soil or material on top of a beam in one bay, or infinite bay (see http://www.facsystems.com/LAGGING.pdf). If bricks are substituted for soil, steel channel for wood lagging, and columns for soldier piles, then it can be concluded that more arching reduction would result as the columns become thicker. Additionally, the shear reduction factor is different than the bending moment reduction factor. The solution in that paper is very precise with less than 2% difference between the results calculated using the reduction factors and those calculated directly. Utilizing the substitutions above, we will explore a confined soil or material, on top of a beam, of which the material ends at a distance *h* above the beam. We will also assume a live load on top of the material. Future research will explore the arching of materials without a beam at the bottom, including wide flange steel beams.

This paper represents the latest iteration of a thought process initiated during my master's Thesis in which I proposed a correction to an error by W. D. Liam Finn, PhD, in his paper, "Boundary Value Problems of Soil Mechanics." My goal was to modify his method to prove the existence of

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arching. The conclusion was discovering the arching factors for wood lagging in shoring walls. There was an error in differentiating the deflection, however, because the deflection, expressed as a Fourier series, was calculated by taking the fourth derivative of the series. This represented an error in the boundary condition because differentiating a Fourier series is not allowed. Furthermore, ignoring the load on top of the pile by taking a Fourier series on the lagging only was incorrect because the load on top of the pile affects the arching and should be considered. Regardless of the error, though, the master's Thesis mathematically proves the existence of arching.

The thesis error was corrected in the article cited above. It is evident from the collocation function method how to arrive at the solution by integrating the stress to equate the deflection. What is not clear, however, is how to utilize the Fourier series representation to obtain the exact deflection. Further research is needed. In the foregoing analysis, the exact solution would be easier to implement than the collocation function.

This paper, for which the master's thesis research and the web article paved the road, will evaluate the infinite bay solution instead of the one bay solution, which are similar when using the Fourier Transform instead of the Fourier series.

Keeping in mind that if there is no deflection, as in rigid beam, then there is no arching and the full load must be taken into account, we start with the phi function in elasticity as a solution to the biharmonic equation:

$$\Phi = \sum_{n=\phi}^{\infty} \frac{1}{\alpha_n^2} \cos \alpha_n x \left[(A + B\alpha_n y) e^{\alpha_n y} + (C + D\alpha_n y) e^{-\alpha_n y} \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; 1$$

Where x and y are the coordinates axis and A, B, C and D are constant and at $n = \phi$ is at $n \rightarrow 0$ but divide by 2 as in the Fourier cosine series first term. α_n is a function of n, $\alpha_n = \frac{n\pi}{b+t}$. Where $2b$ is the bay width and $2t$ is the column width as in Fig. 1. Thus, the stress is

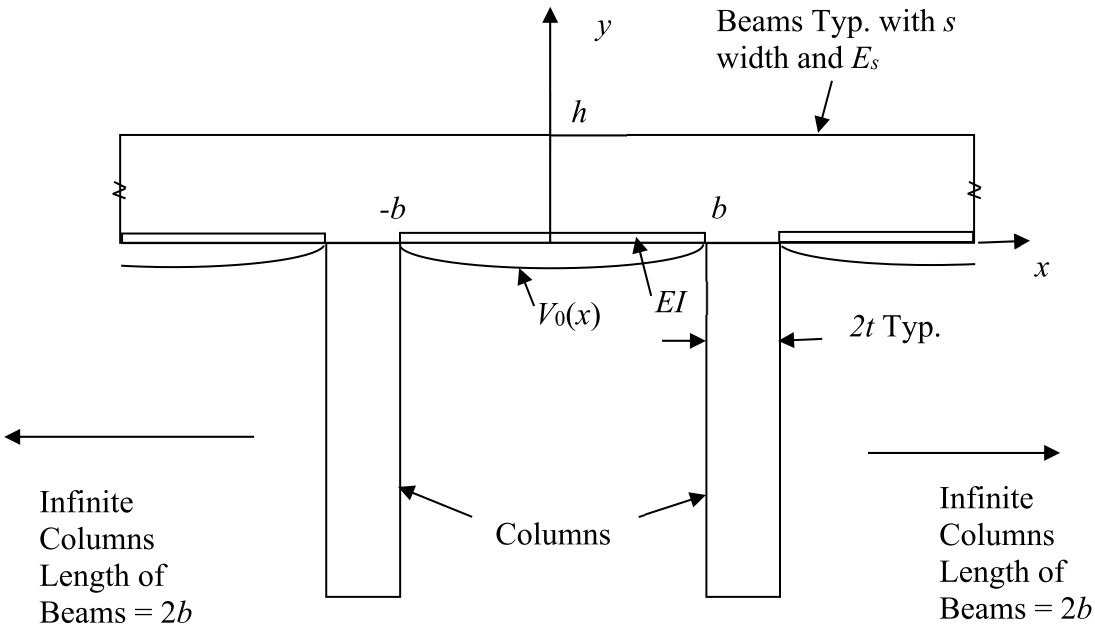


Fig. 1 Beams and Columns Diagram with Bottom Beam

$$\sigma_y = \frac{\partial^2 \Phi}{\partial x^2} = -\sum_{n=\phi}^{\infty} \cos \alpha_n x \left[(A + B\alpha_n y) e^{\alpha_n y} + (C + D\alpha_n y) e^{-\alpha_n y} \right] \quad \ldots \ldots 2$$

At $y = h$ there is the live load $-q$ expressed as a Fourier series = $\sum_{n=\phi}^{\infty} w_n \cos \alpha_n x$ ……………… 3

Thus:

$$(A + B\alpha_n h) e^{\alpha_n h} + (C + D\alpha_n h) e^{-\alpha_n h} = -w_n \quad \ldots \ldots 4$$

Or

$$A = -(C + D\alpha_n h) e^{-2\alpha_n h} - B\alpha_n h - w_n e^{-\alpha_n h} \ldots \ldots 5$$

Now

$$\tau_{xy} = -\frac{\partial^2 \varphi}{\partial x \partial y} = \sum_{n=\varphi}^{\infty} \sin \alpha_n x [(A + B + B\alpha_n y) e^{\alpha_n y} + (-C + D - D\alpha_n y) e^{-\alpha_n y}] \ldots \ldots 6$$

We will assume the shear is zero at $y = h$, thus from Eq. 6 yields:

$(A + B + B\alpha_n h)e^{\alpha_n h} + (-C + D - D\alpha_n h)e^{-\alpha_n h} = 0$ ………………………………………… 7

$\left(-(C + D\alpha_n h)e^{-2\alpha_n h} - B\alpha_n h - w_n e^{-\alpha_n h} + B + B\alpha_n h\right)e^{\alpha_n h} + (-C + D - D\alpha_n h)e^{-\alpha_n h} = 0$

$\left(-(C + D\alpha_n h)e^{-2\alpha_n h} - w_n e^{-\alpha_n h} + B\right)e^{\alpha_n h} + (-C + D - D\alpha_n h)e^{-\alpha_n h} = 0$

$(+B)e^{\alpha_n h} + (-C + D - D\alpha_n h)e^{-\alpha_n h} = (C + D\alpha_n h)e^{-\alpha_n h} + w_n$

$Be^{\alpha_n h} = -(-C + D - D\alpha_n h)e^{-\alpha_n h} + (C + D\alpha_n h)e^{-\alpha_n h} + w_n$

Substituting Eq. 5 in Eq. 7 and solve for *B* yields:

$B = (2C - D + 2D\alpha_n h)e^{-2\alpha_n h} + w_n e^{-\alpha_n h}$ ………………………………………………….. 8

Substitute Eq. 8 in Eq. 5 yields:

$A = -(C + D\alpha_n h)e^{-2\alpha_n h} - \left[(2C - D + 2D\alpha_n h)e^{-2\alpha_n h} + w_n e^{-\alpha_n h}\right]\alpha_n h - w_n e^{-\alpha_n h}$

$A = -\left(C + 2C\alpha_n h + 2D\alpha_n^2 h^2\right)e^{-2\alpha_n h} - w_n(\alpha_n h + 1)e^{-\alpha_n h}$ …………………………………9

We will assume at $y = 0$ the shear is zero, thus from Eq. 6 yields

$A + B - C + D = 0$ ………………………………………………………………………. 10

$$-\left(C + 2C\alpha_n h + 2D\alpha_n^2 h^2\right)e^{-2\alpha_n h} - w_n(\alpha_n h + 1)e^{-\alpha_n h} + (2C - D + 2D\alpha_n h)e^{-2\alpha_n h} + w_n e^{-\alpha_n h} - C + D = 0$$

$$-(C + 2C\alpha_n h)e^{-2\alpha_n h} + 2Ce^{-2\alpha_n h} - C = -D + 2D\alpha_n^2 h^2 e^{-2\alpha_n h} + De^{-2\alpha_n h} - 2D\alpha_n h e^{-2\alpha_n h} + w_n(\alpha_n h + 1)e^{-\alpha_n h} - w_n e^{-\alpha_n h}$$

$$\left[-(1 + 2\alpha_n h)e^{-2\alpha_n h} + 2e^{-2\alpha_n h} - 1\right]C = \left[-1 + 2\alpha_n^2 h^2 e^{-2\alpha_n h} + e^{-2\alpha_n h} - 2\alpha_n h e^{-2\alpha_n h}\right]D + w_n \alpha_n h e^{-\alpha_n h}$$

Substitute Eq. 8 and Eq. 9 in Eq. 10 and solve for *C* yields:

$C = \dfrac{-1 + \left(1 - 2\alpha_n h + 2\alpha_n^2 h^2\right)e^{-2\alpha_n h}}{(1 - 2\alpha_n h)e^{-2\alpha_n h} - 1} D + \dfrac{\alpha_n h e^{-\alpha_n h}}{(1 - 2\alpha_n h)e^{-2\alpha_n h} - 1} w_n$ ………………………………. 11

Let

$C = c_n D + w_{n3} w_n$ ………………………………………………………………… 12

Where

$$c_n = \frac{-1 + \left(1 - 2\alpha_n h + 2\alpha_n^2 h^2\right)e^{-2\alpha_n h}}{(1 - 2\alpha_n h)e^{-2\alpha_n h} - 1} = 1 + \frac{2\alpha_n^2 h^2 e^{-2\alpha_n h}}{(1 - 2\alpha_n h)e^{-2\alpha_n h} - 1}$$

At $h = \infty$ $c_n = 1$

And

$$w_{n3} = \frac{\alpha_n h e^{-\alpha_n h}}{(1-2\alpha_n h)e^{-2\alpha_n h} - 1}$$

Substitute Eq. 12 in Eq. 9 and solve for *A* yields:

$$A = -\left([1+2\alpha_n h][c_n D + w_{n3} w_n] + 2D\alpha_n^2 h^2\right)e^{-2\alpha_n h} - w_n(\alpha_n h + 1)e^{-\alpha_n h}$$

$$A = a_n D + w_{n1} w_n \quad \ldots\ldots 13$$

Where

$$a_n = -\left[c_n(1+2\alpha_n h) + 2\alpha_n^2 h^2\right]e^{-2\alpha_n h}$$

At $h = \infty$ $a_n = 0$

And

$$w_{n1} = -w_{n3}(1+2\alpha_n h)e^{-2\alpha_n h} - (1+\alpha_n h)e^{-\alpha_n h}$$

Substitute Eq. 12 in Eq. 8 yields:

$$B = b_n D + w_{n2} w_n \quad \ldots\ldots 14$$

Where

$$b_n = [2c_n - 1 + 2\alpha_n h]e^{-2\alpha_n h}$$

At $h = \infty$ $b_n = 0$

And

$$w_{n2} = 2w_{n3} e^{-2\alpha_n h} + e^{-\alpha_n h}$$

So, *A*, *B* and *C* are functions of *D*. Now we seek to find *D*.

To find *D* we need to find the deflection, the strain in the *y* direction is:

$$\varepsilon_y = \beta\sigma_y - \rho\sigma_x \quad \ldots\ldots 15$$

Where $\beta = \frac{1-\nu^2}{E_s}$ and $\rho = \frac{\nu(1+\nu)}{E_s}$ for plain strain and $\beta = \frac{1}{E_s}$ and $\rho = \frac{\nu}{E_s}$ for plain stress, ν is Poison's ration and E_s is the elastic modulus of the material on top of the beam and

$$\sigma_y = -\sum_{n=\varphi}^{\infty} cos\,\alpha_n\,x[(A + B\alpha_n y)e^{\alpha_n y} + (C + D\alpha_n y)e^{-\alpha_n y}] - p\left(1 - \frac{y}{h}\right)$$

$$\sigma_y = -\sum_{n=\varphi}^{\infty} cos\,\alpha_n\,x[(a_n D + w_{n1}w_n + (b_n D + w_{n2}w_n)\alpha_n y)e^{\alpha_n y} + (c_n D + w_{n3}w_n + D\alpha_n y)e^{-\alpha_n y}] - p\left(1 - \frac{y}{h}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..16$$

$$\sigma_x = \frac{\partial^2\Phi}{\partial y^2} = \sum_{n=\varphi}^{\infty} cos\,\alpha_n\,x[(A + 2B + B\alpha_n y)e^{\alpha_n y} + (C - 2D + D\alpha_n y)e^{-\alpha_n y}]$$

$$\sigma_x = \sum_{n=\varphi}^{\infty} cos\,\alpha_n\,x[(A + B\alpha_n y)e^{\alpha_n y} + (C + D\alpha_n y)e^{-\alpha_n y}] + 2\sum_{n=\varphi}^{\infty} cos\,\alpha_n\,x[Be^{\alpha_n y} - De^{-\alpha_n y}] - pk_0\left(1 - \frac{y}{h}\right)$$

$$\sigma_x = k_0\sum_{n=\varphi}^{\infty} cos\,\alpha_n\,x[(a_n D + k_0 w_{n1}w_n + (b_n D + k_0 w_{n2}w_n)\alpha_n y)e^{\alpha_n y} + (c_n D + k_0 w_{n3}w_n + D\alpha_n y)e^{-\alpha_n y}] + 2k_0\sum_{n=\varphi}^{\infty} cos\,\alpha_n\,x[(b_n D + k_0 w_{n2}w_n)e^{\alpha_n y} - De^{-\alpha_n y}] - pk_0\left(1 - \frac{y}{h}\right) \ldots\ldots.17$$

$$\tau_{xy} = -\frac{\partial^2\varphi}{\partial x\partial y} = \sum_{n=\varphi}^{\infty} sin\,\alpha_n\,x[(A + B + B\alpha_n y)e^{\alpha_n y} + (-C + D - D\alpha_n y)e^{-\alpha_n y}]$$

$$\tau_{xy} = \sum_{n=\varphi}^{\infty} sin\,\alpha_n\,x[(a_n D + w_{n1}w_n + b_n D + w_{n2}w_n + (b_n D + w_{n2}w_n)\alpha_n y)e^{\alpha_n y} + (-(c_n D + w_{n3}w_n) + D - D\alpha_n y)e^{-\alpha_n y}]$$

$$\tau_{xy} = \sum_{n=\varphi}^{\infty} sin\,\alpha_n\,x[(a_n + b_n)D + w_{n1}w_n + w_{n2}w_n + (b_n D + w_{n2}w_n)\alpha_n y)e^{\alpha_n y} + (-(c_n - 1)D - w_{n3}w_n - D\alpha_n y)e^{-\alpha_n y}] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..18$$

$k_0 = \frac{\rho}{\beta}$ this is because we have the strain ε_x is zero everywhere or the beam has infinite moment of inertia. This is necessary to include the contact shear as described in http://www.facsystems.com/Contact.pdf Where the self-weight being $p\left(1-\frac{y}{h}\right)$. We assume the contact surface of the live load on the mass media is a rough surface. If it is a smooth surface k_0 = 1.0 but for the self-weight $k_0 = \frac{\rho}{\beta}$.

Thus:

$$\varepsilon_y = [-\beta A - \rho(A+2B)]e^{\alpha_n y} + (-\beta-\rho)By\alpha_n e^{\alpha_n y} + [-\beta C - \rho(C-2D)]e^{-\alpha_n y} + (-\beta-\rho)Dy\alpha_n e^{-\alpha_n y} \quad \ldots\ldots\ldots\ldots 19$$

Note: the summation sign and $\cos\alpha_n x$ is not shown for simplicity.

But

$$\varepsilon_y = \frac{\partial V(x)}{\partial y}$$

Then

$$V(x) = [-\beta A - \rho(A+2B)]\frac{e^{\alpha_n y}}{\alpha_n} - (-\beta-\rho)B\frac{e^{\alpha_n y}}{\alpha_n} + (-\beta-\rho)Bye^{\alpha_n y} + [-\beta C - \rho(C-2D)]\frac{e^{-\alpha_n y}}{-\alpha_n} - (-\beta-\rho)D\frac{e^{-\alpha_n y}}{\alpha_n} - (-\beta-\rho)Dye^{-\alpha_n y} + g(x) \quad \ldots\ldots 20$$

At $y = h \to \infty$ and $w_n = 0$, $V(x) = 0$ thus $g(x) = 0$ Note: A, B de-solves the equation to zero.

At $y = 0$ we have:

$$V_0(x) = \sum_{n=\phi}^{\infty} \frac{-\beta(A-B-C-D) - \rho(A+B-C+D)}{\alpha_n}\cos\alpha_n x \quad \ldots\ldots\ldots\ldots 21$$

At $C = D$ and $A = 0$ and $B = 0$ at $h \to \infty$ we have:

$V_0(x) = \sum_{n=\phi}^{\infty} \frac{2\beta D}{\alpha_n}\cos\alpha_n x$ matching the equation in the article at http://www.facsystems.com/LAGGING.pdf.

Substitute Eq. 12, 13 and 14 in Eq. 21 yields:

$$V_0(x) = -\sum_{n=\varphi}^{\infty} \frac{\cos\alpha_n x}{\alpha_n}\{[\beta(a_n - b_n - c_n - 1) + \rho(a_n + b_n - c_n + 1)]D + \beta(w_{n1} - w_{n2} - w_{n3})w_n + \rho(w_{n1} + w_{n2} - w_{n3})w_n\}$$

…… 20

$$V_0(x) = -\sum_{n=\varphi}^{\infty} \frac{\beta\cos\alpha_n x}{\alpha_n}\left\{\left[(a_n - b_n - c_n - 1) + \frac{\rho}{\beta}(a_n + b_n - c_n + 1)\right]D + (w_{n1} - w_{n2} - w_{n3})w_n + \frac{\rho}{\beta}(w_{n1} + w_{n2} - w_{n3})w_n\right\}$$

$$\sum_{n=\varphi}^{\infty} v_n \cos\alpha_n x = -\sum_{n=\varphi}^{\infty} \frac{\beta\cos\alpha_n x}{\alpha_n}\left\{\left[(a_n - b_n - c_n - 1) + \frac{\rho}{\beta}(a_n + b_n - c_n + 1)\right]D + (w_{n1} - w_{n2} - w_{n3})w_n + \frac{\rho}{\beta}(w_{n1} + w_{n2} - w_{n3})w_n\right\}$$

$$-\frac{\alpha_n v_n}{\beta} = \left[(a_n - b_n - c_n - 1) + \frac{\rho}{\beta}(a_n + b_n - c_n + 1)\right]D + (w_{n1} - w_{n2} - w_{n3})w_n + \frac{\rho}{\beta}(w_{n1} + w_{n2} - w_{n3})w_n$$

$$V_0(x) = \sum_{n=\varphi}^{\infty} v_n \cos\alpha_n x$$

where $v_n = \frac{2}{b+t}\int_0^b v_0(\lambda)\cos\alpha_n \lambda d\lambda$ and $v_0(\lambda) = V_0$ is the deflection

Thus

$$D = -\frac{\frac{\alpha_n v_n}{\beta} + (w_{n1} - w_{n2} - w_{n3})w_n + \frac{\rho}{\beta}(w_{n1} + w_{n2} - w_{n3})w_n}{(a_n - b_n - c_n - 1) + \frac{\rho}{\beta}(a_n + b_n - c_n + 1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots..22$$

$\sigma_y @ y = 0$ from Eq. 2 and Eq. 16 yields:

$$\sigma_y = -\sum_{n=\varphi}^{\infty} \cos\alpha_n x(A + C) = -\sum_{n=\varphi}^{\infty} \cos\alpha_n x[(a_n + c_n)D + (w_{n1} + w_{n3})w_n] - p$$

……………………….. 23

Substitute Eq. 22 in 23 yields:

$$\sigma_y = -\sum_{n=\varphi}^{\infty} cos\,\alpha_n\, x \left(\frac{k_n}{\beta}\alpha_n v_n + W_n\right) - p \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..24$$

And p is the self-weight.

Where:

$$k_n = -\frac{a_n + c_n}{d_n}$$

and

$$W_n = -\frac{(a_n + c_n)\left[(w_{n1} - w_{n2} - w_{n3})w_n + \frac{\rho}{\beta}(w_{n1} + w_{n2} - w_{n3})w_n\right]}{(a_n - b_n - c_n - 1) + \frac{\rho}{\beta}(a_n + b_n - c_n + 1)} + (w_{n1} + w_{n3})w_n$$

And

$$d_n = (a_n - b_n - c_n - 1) + \frac{\rho}{\beta}(a_n + b_n - c_n + 1)$$

Now from equation 10

$$A + B - C + D = 0 = a_n D + w_{n1}w_n + b_n D + w_{n2}w_n - c_n D - w_{n3}w_n + D = 0$$

$$a_n + b_n - c_n + 1 = 0$$

And

$$w_{n1} + w_{n2} - w_{n3} = 0$$

Thus

$$D = -\frac{\frac{\alpha_n v_n}{\beta} + (w_{n1} - w_{n2} - w_{n3})w_n}{(a_n - b_n - c_n - 1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..25$$

And

$$W_n = -\frac{(a_n + c_n)(w_{n1} - w_{n2} - w_{n3})w_n}{(a_n - b_n - c_n - 1)} + (w_{n1} + w_{n3})w_n \quad \ldots\ldots\ldots\ldots\ldots\ldots..26$$

And

$$d_n = a_n - b_n - c_n - 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.27$$

Since at $y = 0$

$$\int_0^{b+t} \sigma_y dx = -p(b+t) - q(b+t)$$

Then the $n = 0$ first coefficient drops and n starts at 1 for k_n factor since it does not involve w_n. For example, if $q = 0$, then the equation must hold for all n. We will use a parabola collocation function

$\bar{v}_0(z) = -d_1\delta\,[1-(x/b)^2]$

where $\delta=5sb^4p/(24EI)$ for dead loads, and $\delta=5sb^4q/(24EI)$ for live loads. The charts will be for dead load only and live load only and d_1 is a dimensional constant and will vary with h. Thus,

$$v_n = -\frac{4d_1\delta}{b+t}\left(\frac{1}{b^2\alpha_n^3}\sin\alpha_n b - \frac{1}{b\alpha_n^2}\cos\alpha_n b\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 28$$

It is clear that at $n\rightarrow 0$ $v_n = -\frac{2}{3}d_1\delta b$ and at $n = 0$ in the left term of in Eq. 24 is zero and the first term drops. For the live load:

$$w_n = -\frac{2}{b+t}\int_0^{b+t-l} q\cos\alpha_n\lambda d\lambda = -\frac{2q}{(b+t)\alpha_n}\sin\alpha_n(b+t-l) \ldots\ldots\ldots\ldots\ldots 29$$

$$w_0 = -\frac{q}{b+t}(b+t-l)$$

Rearrange for the stress functions:

$$\sigma_y = -\sum_{n=\varphi}^{\infty}\cos\alpha_n x[(a_nD + w_{n1}w_n + (b_nD + w_{n2}w_n)\alpha_n y)e^{\alpha_n y} + (c_nD + w_{n3}w_n + D\alpha_n y)e^{-\alpha_n y}] - p\left(1-\frac{y}{h}\right)$$

Where:

$$D = -\frac{\alpha_n v_n}{\beta d_n} - \frac{W1_n}{d_n} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 30$$

$$W1_n = (w_{n1} - w_{n2} - w_{n3})w_n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 31$$

The full stress functions will be addressed later, Now at $y = 0$.

where l is a small negligible distance.

Thus, the shear is:

$$\int \sigma_y dx = -\sum_{n=1}^{\infty} sin\, \alpha_n\, x \left(\frac{k_n}{\beta} v_n + \frac{W_n}{\alpha_n}\right) - px \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 32$$

Note: n starts at 1 since W_n as $\alpha_n \rightarrow \infty$ is a finite number. Thus, the shear goes to infinity at $n = 0$, but, since the shear is finite and equal to $p(b+t) + q(b+t)$, $n = 0$ drops. Conversely, the constant of integration will de-solve it to have the shear as an odd function.

The shear reduction for dead load is:

$$R_v = \sum_{n=1}^{\infty} \frac{sin\, \alpha_n\, b}{pb} \left(\frac{k_n}{\beta} v_n\right) + 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 33$$

The shear reduction for live load is:

$$R_v = \sum_{n=1}^{\infty} \frac{\sin \alpha_n\, b}{qb} \left(\frac{k_n}{\beta} v_n + \frac{W_n}{\alpha_n}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 34$$

The moment is

$$\sigma_y dx = \sum_{n=1}^{\infty} cos\, \alpha_n\, x \left(\frac{k_n v_n}{\beta \alpha_n} + \frac{W_n}{\alpha_n^2}\right) - p\frac{x^2}{2} + C1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 35$$

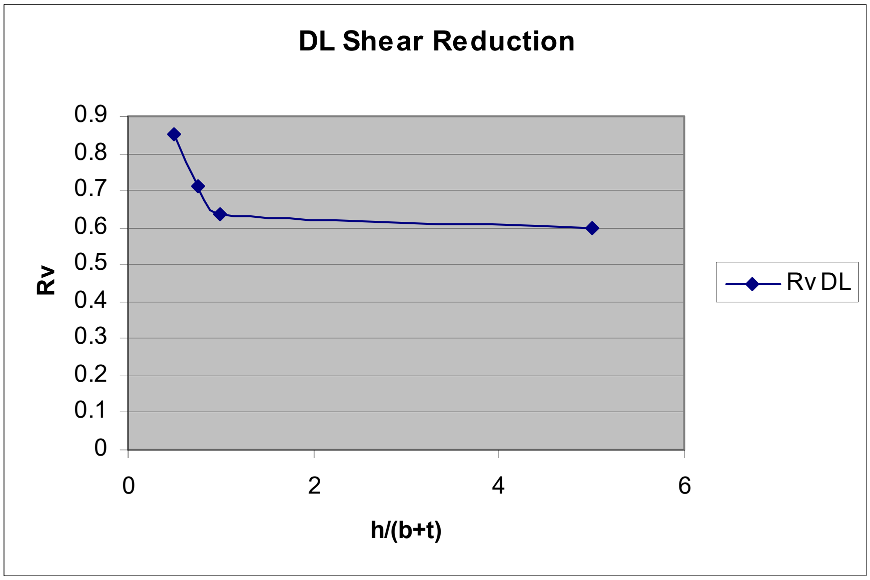
Where $C1$ is a constant of integration, the moment reduction for dead load is:

$$R_m = \sum_{n=1}^{\infty} \frac{(1 - cos\, \alpha_n\, b)}{0.5pb^2} \left(\frac{k_n v_n}{\beta \alpha_n}\right) + 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 36$$

The moment reduction for live load is:

$$R_m = \sum_{n=1}^{\infty} \frac{(1 - cos\, \alpha_n\, b)}{0.5qb^2} \left(\frac{k_n v_n}{\beta \alpha_n} + \frac{W_n}{\alpha_n^2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 37$$

See Fig 2 and 3 for arching reductions



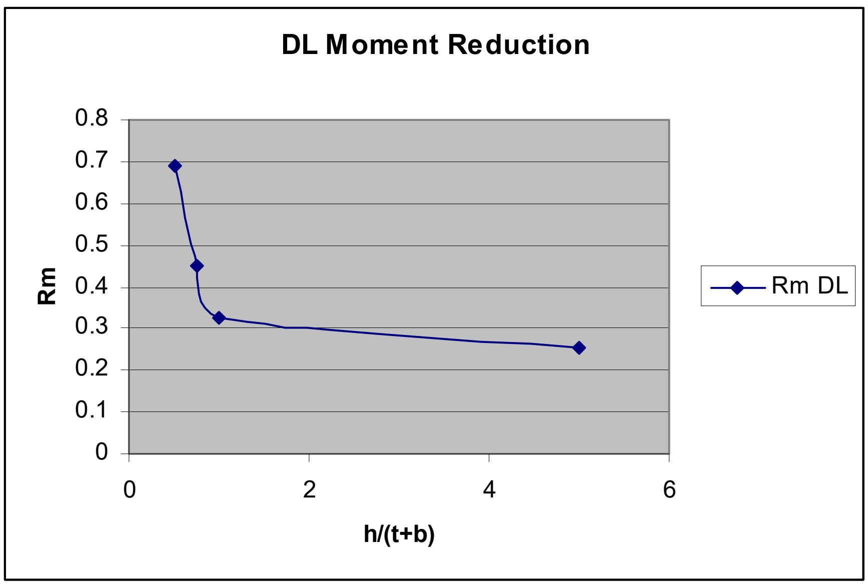


Figure 2 – Two Plots of the Reduction Factors due to Arching for DL

Note:

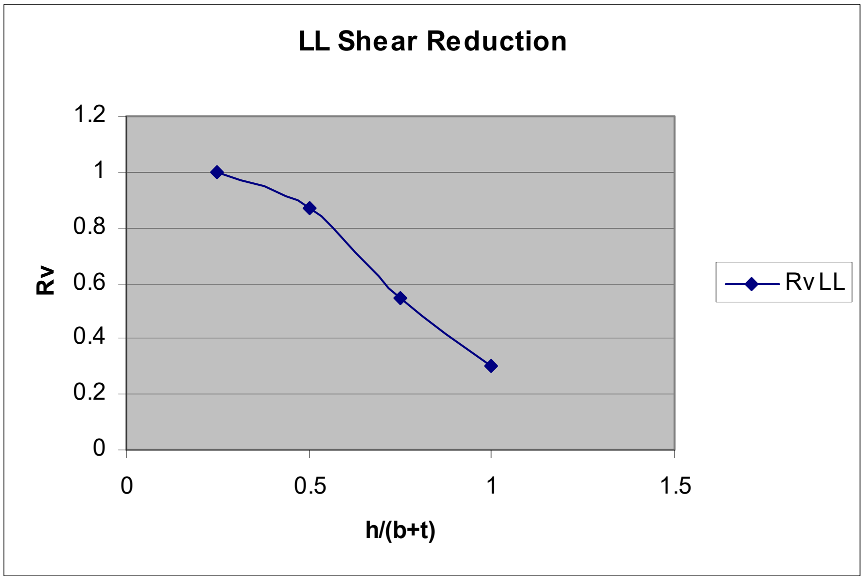
d1 = 0.282667

$$\frac{\delta}{\beta pb} = 6.25$$

b/(b+t) = 0.667

$\nu =$ 0.3 Soils

Note: h/(b+t) will change d1 and it is left constant for the purpose of demonstration
Charts are for demonstration only and not to be used until final charts are derived from research



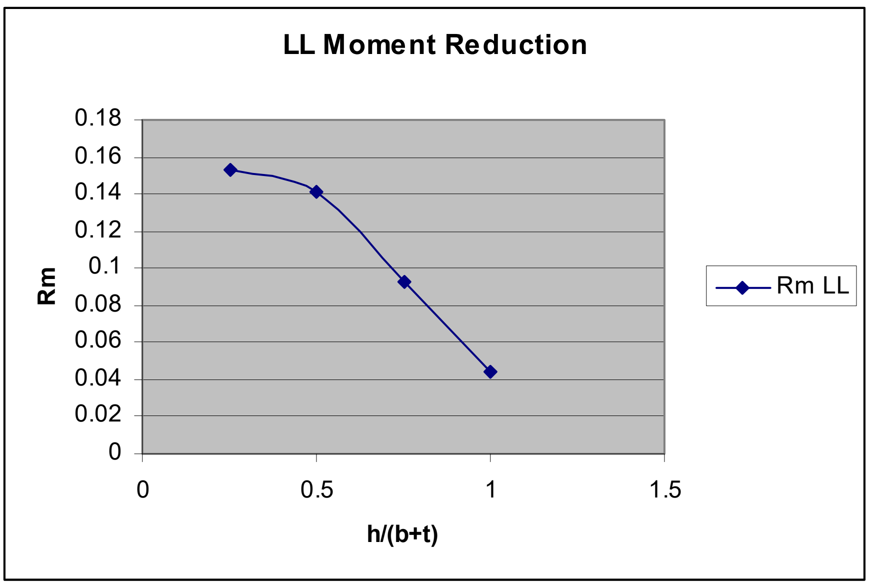


Figure 3 – Two Plots of the Reduction Factors due to Arching for LL

Note:

d1 =	0.01	
$\frac{\delta}{\beta q b} = 6.25$		
b/(b+t) =	0.667	
$\nu =$	0.3	Soils
(b+t-l)/(b+t)=	0.85	

Note: h/(b+t) will change d1 and it is left constant for the purpose of demonstration
Charts are for demonstration only and not to be used until final charts are derived from research

Figure 4 shows the mechanics underling the loading.

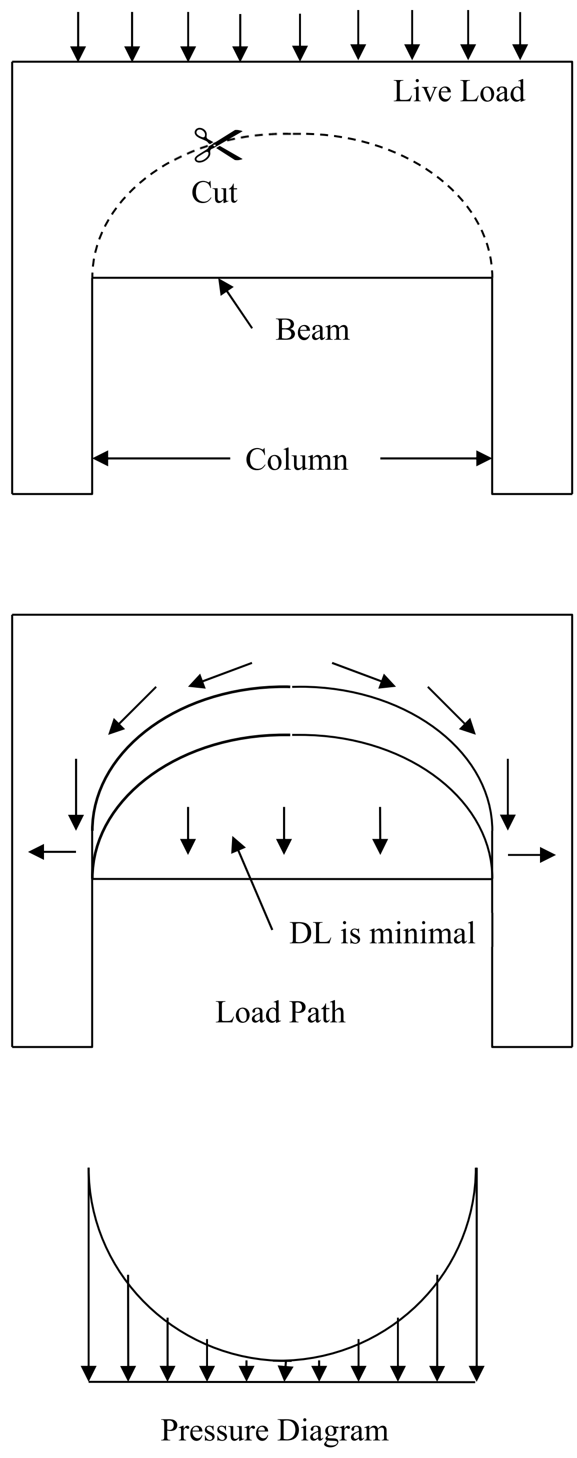


Figure 4 – When cut to an arch the load path and pressure diagram is exposed
If the removed portion is put back the arch does not disappear

Closed form solution #1 simple span lagging with $h = \infty$:

Starting with at $C = D$ and $A = 0$ and $B = 0$ at $h \rightarrow \infty$ we have:

$V_0(x) = \sum_{n=\phi}^{\infty} \frac{2\beta D}{\alpha_n} \cos \alpha_n x$ matching the equation in the article at

http://www.facsystems.com/LAGGING.pdf

We have the stress and reaction yields:

$$\sigma_y\big|_{y=0} = -\sum_{n=1}^{\infty} \frac{s\alpha_n v_n}{2\beta} \cos \alpha_n x - p \qquad -b \le x \le b \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots 38$$

$$\sigma_y\big|_{y=0} = -\sum_{n=1}^{\infty} \frac{s\alpha_n v_n}{2\beta} \cos \alpha_n x + p\frac{b}{t} \qquad b \le x \le b + t \quad \dots.\dots.\dots.39$$

In the past we forced the deflection under the column to be zero which gave infinite stress at the corners or at $x = \pm\, b$. In here we do it from stress requirements. It can be seen by setting 1/β =0 or water the stress it $-p$ at the lagging in the region $-b \le x \le b$ and pb/t reaction at the column. This is necessary so the summation of forces to be zero. Where, $-p$ is in lb/ft and the stress is in strip s = width of lagging board in the z-direction or thickness of the plate. The reaction at the column is portrait as positive it does not mean tension the final solution at the column must be reversed or multiplied by -1 to show compression at the column resulting in increased stress at the corners. This will limit the solution since when t/b is large the stress at the column from the lagging is limited to the bearing area at the corners of the column and does not approach zero. However, the stress has an average of pb/t so the stress at the column is not uniform we assume smaller t. The stress $-p$ at the column is not shown since the reaction will cancel it out with p.

It was found from the above article that at $t/b > 0.5$ the deflection at the column can be zero for all $0.5 < t/b < \infty$ and the solution is that of $t/b = 0.5$ can be used as a conservative solution $t/b > 0.5$. The moment reduction its around 2.2% over and 15.5% under for shear reduction this percentage will be found in the following solution. The stress at $x = 0$ match very closely $A = 16.53$ verses 16.5 gives zero pressure. The reason of this difference is the stress in the past article at $x = b$ is infinitely negative or compressive and does not have a value but the stress. However, in the new solution it has a value and makes more sense and the deflection at the column corners is not quite zero. So, I would trust the new following solution. Note at $x = 0$ both stress and moment are very close in both solutions.

The shear yields:

$$\int \sigma_y = -\sum_{n=1}^{\infty} \frac{s v_n}{2\beta} \sin \alpha_n x - px \qquad -b \le x \le b \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots..40$$

$$\int \sigma_y = -\sum_{n=1}^{\infty} \frac{s v_n}{2\beta} \sin \alpha_n x + p\frac{b}{t}(x-b-t) \qquad b \le x \le b+t \text{ ....41}$$

The coefficient of integration is zero since the shear is zero at $x = 0$. We can rewrite in Fourier sine series in the interval $-b-t \leq x \leq b+t$ where the line in the left side of the equation has an active shear with slope $-p$ in $-b \leq x \leq b$ and a reactive shear at the columns with a slope pb/t for equilibrium. So, when differentiating over the entire x domain we obtain active pressure $-p$ for arching and a reactive pressure pb/t for the column reaction. If $b = t$ there is a load $-p$ at the column there is a reaction equal p for that load, and they cancel each other out. This is necessary allowing downward deflection at lagging and upward deflection at columns. Thus, when multiplying the stress by the distances to obtain the forces in the y direction the summation of forces is zero see Figure 5, yields

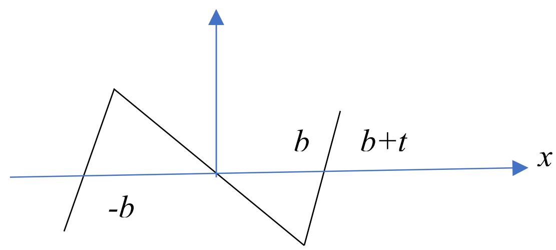


Figure 5, Saw tooth shear or rotation

$$\text{the saw tooth is} = -\frac{2p}{t}\sum_{n=1}^{\infty}\frac{\sin \alpha_n b}{\alpha_n^2}\sin \alpha_n x$$

$$\int \sigma_y = -\sum_{n=1}^{\infty} \frac{s v_n}{2\beta} \sin \alpha_n x - \frac{2p}{t}\sum_{n=1}^{\infty}\frac{\sin \alpha_n b}{\alpha_n^2}\sin \alpha_n x \text{ ....42}$$

The moment yields

$$\iint \sigma_y = \sum_{n=1}^{\infty} \frac{s v_n}{2\beta\, \alpha_n} \cos \alpha_n x + \frac{2p}{t}\sum_{n=1}^{\infty}\frac{\sin \alpha_n b}{\alpha_n^3}\cos \alpha_n x + C2 \text{ ....43}$$

When vn = 0 or $\beta = \infty$ as in water then at $x = b$ the moment is zero and

$$C2 = -\frac{2p}{t}\sum_{n=1}^{\infty}\frac{\sin \alpha_n b}{\alpha_n^3}\cos \alpha_n b = -p\frac{b^2}{2} \;\; in -b \le x \le b \text{ ....44}$$

The rotation yields

$$\frac{1}{EI}\iiint \sigma_y = \frac{1}{EI}\left[\sum_{n=1}^{\infty}\frac{sv_n}{2\beta\,\alpha_n^2}\sin\alpha_n x + \frac{2p}{t}\sum_{n=1}^{\infty}\frac{\sin\alpha_n b}{\alpha_n^4}\sin\alpha_n x + C2x\right] \dots\dots\dots\dots\dots..45$$

The coefficient of integration is zero since the rotation slope is zero at *x* = 0. We can rewrite in Fourier sine series in the interval $-b-t \le x \le b+t$ where the line *C2x* has an active slope with moment C2 in $-b \le x \le b$ and a reactive slope at the columns with a moment -C2 *b/t* so there is a finite rotation value at $x = \pm b$ and the rotation at $x = t$ is C2*b/t* x *t* = C2*b* under the column and it is a saw tooth function. This is necessary for compatibility when the moment of inertia is the same for the beam and the column allowing downward deflection at lagging and upward deflection at columns such that the involuntary parabolic deflection of the moment C2 in $-b \le x \le b$ is met with a reverse parabolic deflection in $-b - 2t \le x \le -b$ and $b \le x \le b+2t$ and so on giving a wavy deflection with compatibility slope at $b = t$. Also, when differentiating over the entire *x* domain we obtain active moment C2 for arching and a reactive moment for the column -C2 when $b = t$ and have equilibrium. Part of the reason of doing the saw tooth because in equation **24** we restricted the deflection to be zero at the columns where in here we did not we let it be controlled by the moment on inertia, see Figure 5, yields.

$$\iiint \sigma_y = \frac{1}{EI}\left[\sum_{n=1}^{\infty}\frac{sv_n}{2\beta\,\alpha_n^2}\sin\alpha_n x + \frac{2p}{t}\sum_{n=1}^{\infty}\frac{\sin\alpha_n b}{\alpha_n^4}\sin\alpha_n x + \frac{2C2}{t}\sum_{n=1}^{\infty}\frac{\sin\alpha_n b}{\alpha_n^2}\sin\alpha_n x\right] \dots.46$$

In here *I* is the same in the region $-b \le x \le b$ then of the region $-b - 2t \le x \le -b$ and $b \le x \le b+2t$ the columns have large moment of inertia and the rotation and deflection is practically zero in that region when the deflection is derived it will be shown it is practically zero at the column when the rotation compatibility is not required. This is also true when *t* is infinite the slope is zero in that region under the column with 1/β =0 or water. We leave the solution generic knowing the solution in each region is different and require substituting the proper moment of inertia. This solution will not affect the deflection at the lagging and does not change the moments, shear and stress since we require the moment to be zero at $x = \pm b$.

Finally, the deflection yields

$$\sum_{n=1}^{\infty} v_n\, cos\,\alpha_n x + \frac{v_0}{2} = \frac{1}{EI}\left[-\sum_{n=1}^{\infty}\frac{sv_n}{2\beta\,\alpha_n^3}\cos\alpha_n x - \frac{2p}{t}\sum_{n=1}^{\infty}\frac{\sin\alpha_n b}{\alpha_n^5}\cos\alpha_n x - \frac{2C2}{t}\sum_{n=1}^{\infty}\frac{\sin\alpha_n b}{\alpha_n^3}\cos\alpha_n x + C4\right] \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.47$$

C4 and v0/2 cancel and solve for *vn* yields:

$$v_n\left[EI+\frac{s}{2\beta\ \alpha_n^3}\right]=-\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^5}-\frac{2C2}{t}\frac{\sin\alpha_n b}{\alpha_n^3}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.48$$

Or

$$v_n=-\frac{\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^5}+\frac{2C2}{t}\frac{\sin\alpha_n b}{\alpha_n^3}}{EI+\frac{s}{2\beta\ \alpha_n^3}}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 49$$

$$\frac{C4}{EI}=\frac{v_0}{2}=\frac{1}{2}\lim_{n=0}-\frac{\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^2}+\frac{2C2}{t}\sin\alpha_n b}{EI\alpha_n^3+\frac{s}{2\beta}}=-\lim_{n=0}\frac{2p\beta}{ts}\ \frac{\frac{\pi b}{b+t}}{\frac{\pi^2}{(b+t)^2}}\frac{\cos\alpha_n b}{2n}=-\frac{p\beta(b+t)b}{\pi tsn}$$

$$=\ -\frac{p2\beta EI(b+t)b^4}{2\pi tsb^3EIn}=\ -\frac{p(b+t)b^4}{2\pi tAEIn}=\infty\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..50$$

$$A=\frac{sb^3}{2\beta EI}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.51$$

This can happen if the deflection is not defined at $x = \pm b$. So, we set n to start from 1 and v0 is found from C4 when the deflection set equal zero at $\pm b$.

β is in 1/KSI and p is Kip/inch thus v0 dimension is in inch. Substitute vn in the moment

$$\iint\sigma_y=-\sum_{n=1}^{\infty}\frac{s}{2\beta\ \alpha_n}\frac{\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^5}+\frac{2C2}{t}\frac{\sin\alpha_n b}{\alpha_n^3}}{EI+\frac{s}{2\beta\ \alpha_n^3}}\cos\alpha_n x+\frac{2p}{t}\sum_{n=1}^{\infty}\frac{\sin\alpha_n b}{\alpha_n^3}\cos\alpha_n x+C2\ \ldots 52$$

Or

$$\iint\sigma_y=-\sum_{n=1}^{\infty}\frac{\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^3}+\frac{2C2}{t}\frac{\sin\alpha_n b}{\alpha_n}}{\frac{2\beta EI\ a_n^3}{s}+1}\cos\alpha_n x+\frac{2p}{t}\sum_{n=1}^{\infty}\frac{\sin\alpha_n b}{\alpha_n^3}\cos\alpha_n x+C2\ \ldots\ldots\ldots.53$$

At $x = b$ the moment is zero and solve for C2 and the closed from solution is obtained.

$$0=-\sum_{n=1}^{\infty}\frac{\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^3}}{\frac{2\beta EI\ a_n^3}{s}+1}\cos\alpha_n b+\frac{2p}{t}\sum_{n=1}^{\infty}\frac{\sin\alpha_n b}{\alpha_n^3}\cos\alpha_n b+C2$$

$$-C2\sum_{n=1}^{\infty}\frac{\frac{2}{t}\frac{\sin\alpha_n b}{\alpha_n}}{\frac{2\beta EI\ a_n^3}{s}+1}\cos\alpha_n b\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\ldots\ldots 54$$

$$C2=\frac{-\sum_{n=1}^{\infty}\frac{\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^3}}{\frac{2\beta EI\ a_n^3}{s}+1}\cos\alpha_n b+\frac{2p}{t}\sum_{n=1}^{\infty}\frac{\sin\alpha_n b}{\alpha_n^3}\cos\alpha_n b}{\sum_{n=1}^{\infty}\frac{\frac{2}{t}\frac{\sin\alpha_n b}{\alpha_n}}{\frac{2\beta EI\ a_n^3}{s}+1}\cos\alpha_n b-1}\ \ldots\ldots\ldots..55$$

Or

$$C2=\frac{\sum_{n=1}^{\infty}\frac{\frac{2\beta EI\ a_n^3}{s}\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^3}}{\frac{2\beta EI\ a_n^3}{s}+1}\cos\alpha_n b}{\sum_{n=1}^{\infty}\frac{\frac{2}{t}\frac{\sin\alpha_n b}{\alpha_n}}{\frac{2\beta EI\ a_n^3}{s}+1}\cos\alpha_n b-1}=\frac{\sum_{n=1}^{\infty}\frac{\frac{2\beta EI}{s}\frac{2p}{t}\sin\alpha_n b}{\frac{2\beta EI\ a_n^3}{s}+1}\cos\alpha_n b}{\sum_{n=1}^{\infty}\frac{\frac{2}{t}\frac{\sin\alpha_n b}{\alpha_n}}{\frac{2\beta EI\ a_n^3}{s}+1}\cos\alpha_n b-1}\ \ldots\ldots..56$$

$$\frac{C2}{0.5pb^2}=\frac{\sum_{n=1}^{\infty}\frac{\frac{2\beta EI}{sb^3}\frac{4b}{t}\sin\alpha_n b}{\frac{2\beta EI\ b^3a_n^3}{sb^3}+1}\cos\alpha_n b}{\sum_{n=1}^{\infty}\frac{\frac{2}{t}\frac{\sin\alpha_n b}{\alpha_n}}{\frac{2\beta EI\ b^3a_n^3}{sb^3}+1}\cos\alpha_n b-1}=\frac{\sum_{n=1}^{\infty}\frac{\frac{4b}{t}\sin\alpha_n b}{b^3a_n^3+A}\cos\alpha_n b}{A\sum_{n=1}^{\infty}\frac{\frac{2b}{t}\frac{\sin\alpha_n b}{b\alpha_n}}{b^3a_n^3+A}\cos\alpha_n b-1}\ \ldots\ldots.57$$

$$\frac{C2}{0.5pb^2}=\frac{\sum_{n=1}^{\infty}\frac{\frac{4}{\Gamma}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3}+A}\cos\alpha_n b}{A\sum_{n=1}^{\infty}\frac{2\frac{(1+\Gamma)}{\Gamma}\frac{\sin\alpha_n b}{\pi n}}{\frac{(\pi n)^3}{(1+\Gamma)^3}+A}\cos\alpha_n b-1}=\frac{\sum_{n=1}^{\infty}\frac{4\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3}+A}\cos\alpha_n b}{A\sum_{n=1}^{\infty}\frac{2(1+\Gamma)\frac{\sin\alpha_n b}{\pi n}}{\frac{(\pi n)^3}{(1+\Gamma)^3}+A}\cos\alpha_n b-\Gamma}$$

Finally, with Γ= *t/b*:

$$\frac{C2}{0.5pb^2} = \frac{\sum_{n=1}^{\infty} \frac{\frac{4 \sin \alpha_n b}{(\pi n)^3}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + A} \cos \alpha_n b}{A \sum_{n=1}^{\infty} \frac{2(1+\Gamma) \frac{\sin \alpha_n b}{\pi n}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + A} \cos \alpha_n b - \Gamma} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 58$$

The following solution is in the region $-b \leq x \leq b$

The Moment becomes at $x = 0$:

Substitute vn in the shear we have,

$$M = \iint \sigma_y = -\sum_{n=1}^{\infty} \frac{\frac{2p}{t} \frac{\sin \alpha_n b}{\alpha_n^3} + \frac{2C2}{t} \frac{\sin \alpha_n b}{\alpha_n}}{\frac{2\beta EI\, \alpha_n^3}{s} + 1} + \frac{2p}{t} \sum_{n=1}^{\infty} \frac{\sin \alpha_n b}{\alpha_n^3} + C2 \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 59$$

$$\frac{M}{0.5pb^2} = Rm = -\sum_{n=1}^{\infty} \frac{\frac{4b}{t} \frac{\sin \alpha_n b}{b^3 \alpha_n^3} + \frac{2C2b}{0.5b^2 t} \frac{\sin \alpha_n b}{b \alpha_n}}{\frac{2\beta EI\, \alpha_n^3}{s} + 1} + \frac{4b}{t} \sum_{n=1}^{\infty} \frac{\sin \alpha_n b}{b^3 \alpha_n^3} + \frac{C2}{0.5b^2} \quad \ldots \ldots \ldots \ldots \ldots 60$$

The Moment reduction becomes

$$Rm = \sum_{n=1}^{\infty} \frac{\frac{4b}{t} \frac{2\beta EI\, \alpha_n^3}{s} \frac{\sin \alpha_n b}{b^3 \alpha_n^3} - \frac{2C2b}{0.5pb^2 t} \frac{\sin \alpha_n b}{b \alpha_n}}{\frac{2\beta EI\, \alpha_n^3}{s} + 1} + \frac{C2}{0.5pb^2} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 61$$

$$Rm = \sum_{n=1}^{\infty} \frac{\frac{4b}{t} \frac{2\beta EI}{s} \frac{\sin \alpha_n b}{b^3} - \frac{2C2b}{0.5pb^2 t} \frac{\sin \alpha_n b}{b \alpha_n}}{\frac{2\beta EI\, \alpha_n^3}{s} + 1} + \frac{C2}{0.5pb^2} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots .62$$

$$Rm = \sum_{n=1}^{\infty} \frac{\frac{4}{\Gamma} \sin \alpha_n b - \frac{2AC2}{0.5pb^2 \Gamma} \frac{\sin \alpha_n b}{b \alpha_n}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + A} + \frac{C2}{0.5pb^2} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots ..63$$

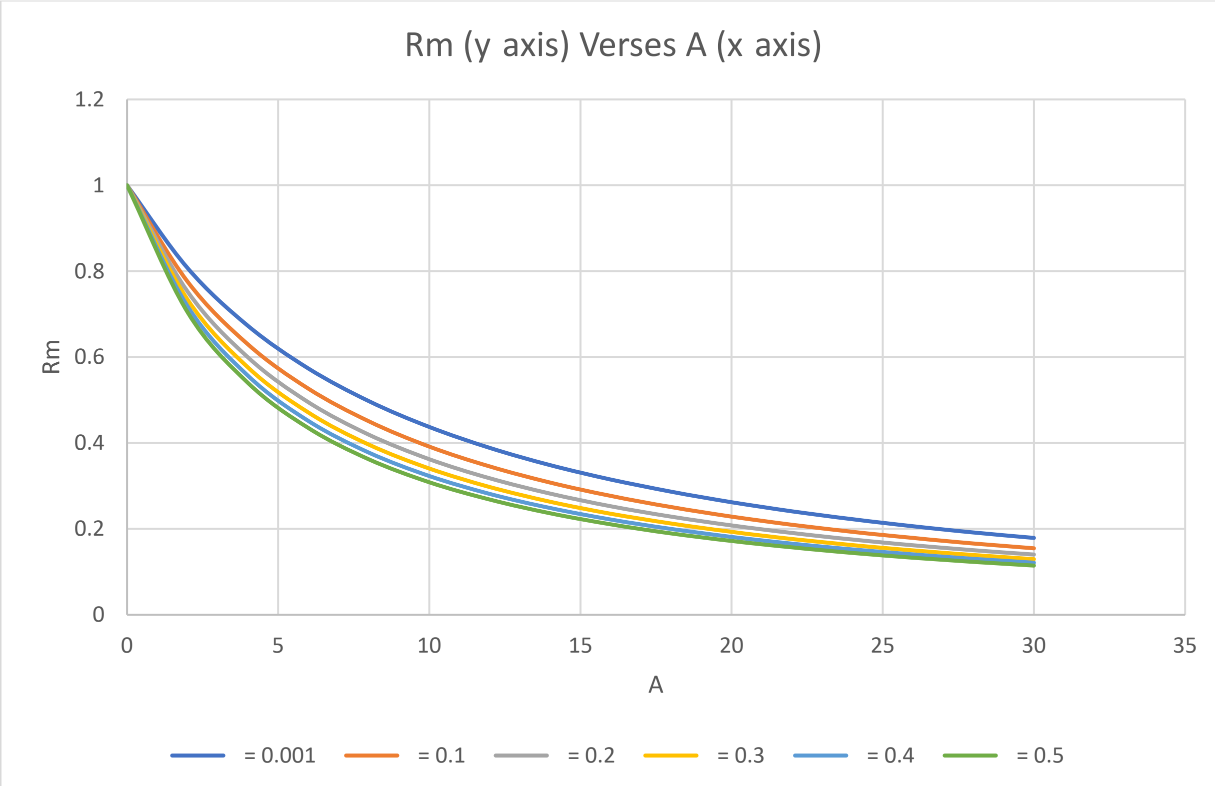
Finally, the Moment reduction at $x = 0$ becomes

$$Rm = \frac{1}{\Gamma}\sum_{n=1}^{\infty}\frac{4\sin\alpha_n b - 2A\left(\frac{C2}{0.5pb^2}\right)\frac{(1+\Gamma)}{\pi n}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3} + A} + \frac{C2}{0.5pb^2} \quad \dots\dots\dots\dots\dots\dots 64$$

Note: the maximum moment may not be at $x = 0$ depending on A

Rm Table 1 for different t/b

A	= 0.001	= 0.1	= 0.2	= 0.3	= 0.4	= 0.5
0	1	1	1	1	1	1
2	0.807309	0.775713	0.752687	0.734192	0.718501	0.704682
4	0.672642	0.629316	0.599158	0.575732	0.556381	0.539728
6	0.573373	0.526407	0.494745	0.470694	0.45117	0.434613
8	0.49728	0.450234	0.41925	0.396083	0.3775	0.361901
10	0.437181	0.391665	0.36221	0.340437	0.323123	0.308697
12	0.388581	0.345297	0.317659	0.297405	0.281402	0.268142
14	0.348521	0.307729	0.281952	0.263184	0.248426	0.236251
16	0.314975	0.276715	0.252731	0.235355	0.221743	0.210551
18	0.286509	0.250709	0.228407	0.212311	0.199737	0.189426
20	0.26208	0.228617	0.20787	0.192939	0.1813	0.171777
22	0.240909	0.209639	0.190319	0.176445	0.165647	0.156827
24	0.222407	0.193178	0.175164	0.162248	0.152207	0.144017
26	0.206116	0.17878	0.16196	0.149911	0.140553	0.132928
28	0.191678	0.166093	0.150364	0.139103	0.130361	0.123246
30	0.178805	0.15484	0.140109	0.129565	0.121382	0.114727



Substitute vn in the shear

$$\int \sigma_y = \sum_{n=1}^{\infty} \frac{\frac{2p}{t}\frac{\sin \alpha_n b}{\alpha_n^2} + \frac{2C2}{t}\sin \alpha_n b}{\frac{2\beta EI\ a_n^3}{s} + 1} sin\ \alpha_n\ x - \frac{2p}{t}\sum_{n=1}^{\infty} \frac{\sin \alpha_n b}{\alpha_n^2} \sin \alpha_n x \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots .65$$

The shear reduction becomes at $x = b$:

$$\frac{V}{-pb} = Rv = -\sum_{n=1}^{\infty} \frac{\frac{2b}{t}\frac{\sin \alpha_n b}{\alpha_n^2 b^2} + \frac{2bC2}{tpb^2}\sin \alpha_n b}{\frac{2\beta EI\ b^3 a_n^3}{sb^3} + 1} sin\ \alpha_n\ b + \frac{2b}{t}\sum_{n=1}^{\infty} \frac{\sin \alpha_n b}{\alpha_n^2 b^2} \sin \alpha_n b\ \ \ldots\ldots .66$$

$$\frac{V}{-pb} = Rv = -\sum_{n=1}^{\infty} \frac{\frac{2b}{t}\frac{\sin \alpha_n b}{\alpha_n^2 b^2} + \frac{2bC2}{tpb^2}\sin \alpha_n b}{\frac{2\beta EI\ b^3 a_n^3}{sb^3} + 1} sin\ \alpha_n\ b + \frac{2b}{t}\sum_{n=1}^{\infty} \frac{\sin \alpha_n b}{\alpha_n^2 b^2} \sin \alpha_n b \ldots\ldots\ldots 67$$

$$Rv = -A\sum_{n=1}^{\infty}\frac{\frac{2}{\Gamma}(1+\Gamma)^2\frac{\sin\alpha_n b}{(\pi n)^2}+\frac{2C2}{\Gamma p b^2}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3}+A} sin\,\alpha_n\, b + \frac{2}{\Gamma}(1+\Gamma)^2\sum_{n=1}^{\infty}\frac{\sin\alpha_n b}{(\pi n)^2}\sin\alpha_n b$$

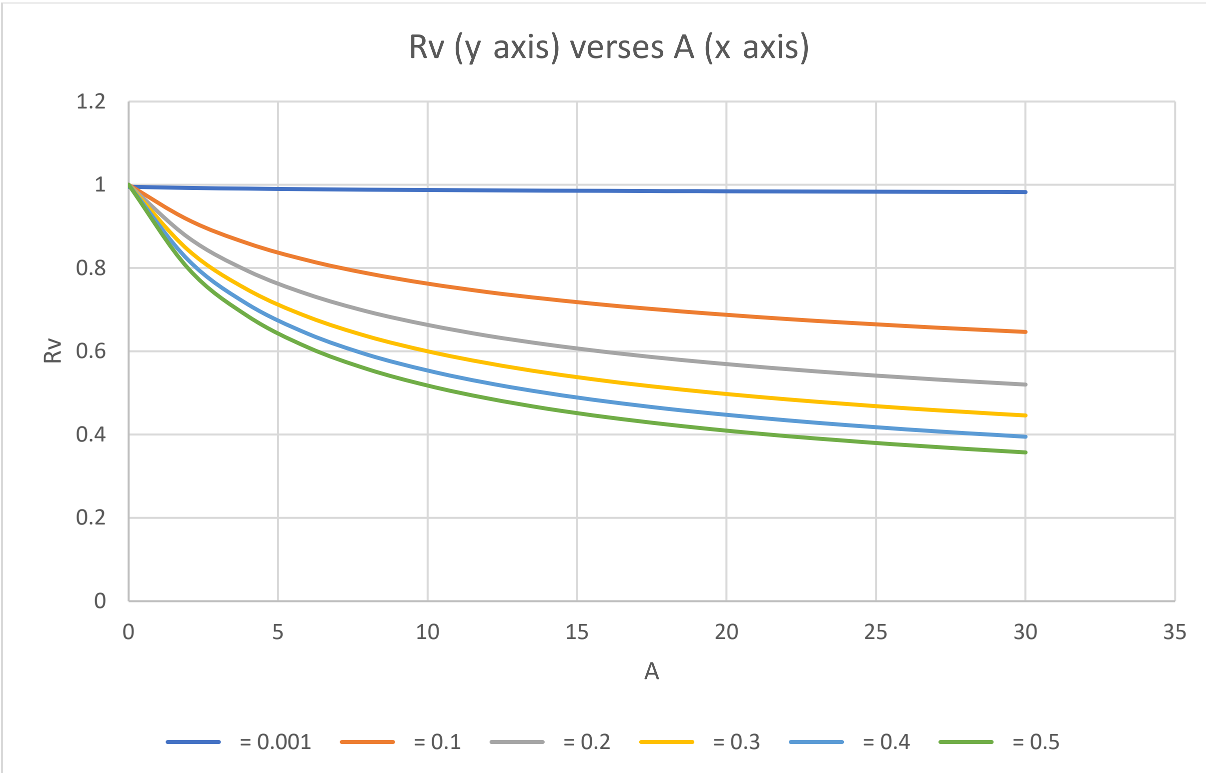
$$Rv = -A\sum_{n=1}^{\infty}\frac{\frac{2}{\Gamma}(1+\Gamma)^2\frac{\sin\alpha_n b}{(\pi n)^2}+\frac{2C2}{\Gamma p b^2}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3}+A} sin\,\alpha_n\, b$$
$$+\frac{2A}{\Gamma}(1+\Gamma)^2\frac{\sin\alpha_n b}{(\pi n)^2}\sin\alpha_n b\frac{1}{\frac{(\pi n)^3}{(1+\Gamma)^3}+A}$$
$$+\frac{2}{\Gamma}(1+\Gamma)^2\frac{(\pi n)^3}{(1+\Gamma)^3}\frac{\sin\alpha_n b}{(\pi n)^2}\sin\alpha_n b\frac{1}{\frac{(\pi n)^3}{(1+\Gamma)^3}+A} \ldots\ldots\ldots..68$$

Finally, the shear reduction at $x = b^*$ becomes

$$Rv = \frac{1}{\Gamma}\sum_{n=1}^{\infty}\frac{2\frac{\pi n}{(1+\Gamma)}-A\frac{C2}{0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3}+A}\sin\alpha_n b\sin\alpha_n b^* \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 69$$

Rv Table 2 for different t/b b=b*

A	= 0.001	= 0.1	= 0.2	= 0.3	= 0.4	= 0.5
0	0.9949295	0.9999387	0.9999635	0.9999715	0.9999752	0.9999772
2	0.9924688	0.9153313	0.8729995	0.8426343	0.8185008	0.7981535
4	0.9906877	0.8588505	0.7926728	0.7472143	0.7122938	0.68369
6	0.9893228	0.8181177	0.7368287	0.6826747	0.6420409	0.6094085
8	0.9882319	0.787103	0.6954463	0.6357757	0.5917738	0.5569488
10	0.9873312	0.7625172	0.6633309	0.5999157	0.5537789	0.5176781
12	0.9865682	0.7424127	0.6375201	0.5714337	0.5238727	0.4870007
14	0.9859085	0.7255623	0.6162	0.5481348	0.4995886	0.4622441
16	0.9853283	0.7111541	0.5981976	0.5286231	0.4793775	0.4417476
18	0.9848108	0.6986291	0.5827204	0.5119674	0.462217	0.4244241
20	0.9843438	0.6875899	0.5692132	0.4975229	0.4474048	0.4095321
22	0.983918	0.6777457	0.5572753	0.4848283	0.434442	0.3965477
24	0.9835265	0.668879	0.5466101	0.4735448	0.4229646	0.3850901
26	0.9831639	0.6608234	0.5369933	0.463418	0.4127001	0.374876
28	0.9828259	0.6534498	0.5282518	0.4542525	0.4034407	0.3656896
30	0.982509	0.6466561	0.52025	0.4458963	0.3950249	0.3573639



Again, when I is very large or infinite, we get the saw tooth shear and we have equilibrium. The stress is

$$\sigma_y = \sum_{n=1}^{\infty} \frac{\frac{2p}{t}\frac{\sin \alpha_n b}{\alpha_n} + 2C2\alpha_n \frac{\sin \alpha_n b}{t}}{\frac{2\beta EI\, a_n^3}{s} + 1} cos\, \alpha_n\, x - \frac{2p}{t}\sum_{n=1}^{\infty} \frac{\sin \alpha_n b}{\alpha_n} \cos \alpha_n x \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots 70$$

Again, when I is very large or infinite we get the step stress $-p$ and pb/t necessary stress for equilibrium on the lagging and columns. At $x = 0$ the stress is

$$\sigma_y = \sum_{n=1}^{\infty} \frac{\frac{2p}{t}\frac{\sin \alpha_n b}{\alpha_n} + 2C2\alpha_n \frac{\sin \alpha_n b}{t}}{\frac{2\beta EI\, a_n^3}{s} + 1} - \frac{2p}{t}\sum_{n=1}^{\infty} \frac{\sin \alpha_n b}{\alpha_n} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\,.\,.\,71$$

Or

$$\frac{\sigma_y}{p} (at\ y = 0\ and\ x = 0) = \frac{sft}{p} = \frac{A}{\Gamma}\sum_{n=1}^{\infty} \frac{2\frac{(1+\Gamma)}{\pi n} + \frac{\pi n}{(1+\Gamma)}\frac{C2}{0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + A} \sin \alpha_n b - 1 \ldots\ldots 72$$

Where ft is the tension in the center of lagging. When A is large, we rewrite from the shear:

$$\frac{sft}{p} = \frac{1}{\Gamma}\sum_{n=1}^{\infty} \frac{2\frac{\pi n}{(1+\Gamma)} - A\frac{C2}{0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + A} \frac{\pi n}{(1+\Gamma)} \sin \alpha_n b \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .73$$

$$\frac{sft}{p} = \frac{1}{\Gamma}\sum_{n=1}^{\infty} -\frac{C2}{0.5pb^2}\frac{\pi n}{(1+\Gamma)} \sin \alpha_n b = 0 \quad \text{as} \quad \frac{C2}{0.5pb^2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots..74$$

Or when the solid is rigid the stress is zero. The stress becomes tension for some *A* but when *A* is large there is zero stress. Thus *sft/p* has a maximum positive tension value see table below. As when *A* = 0 its water.

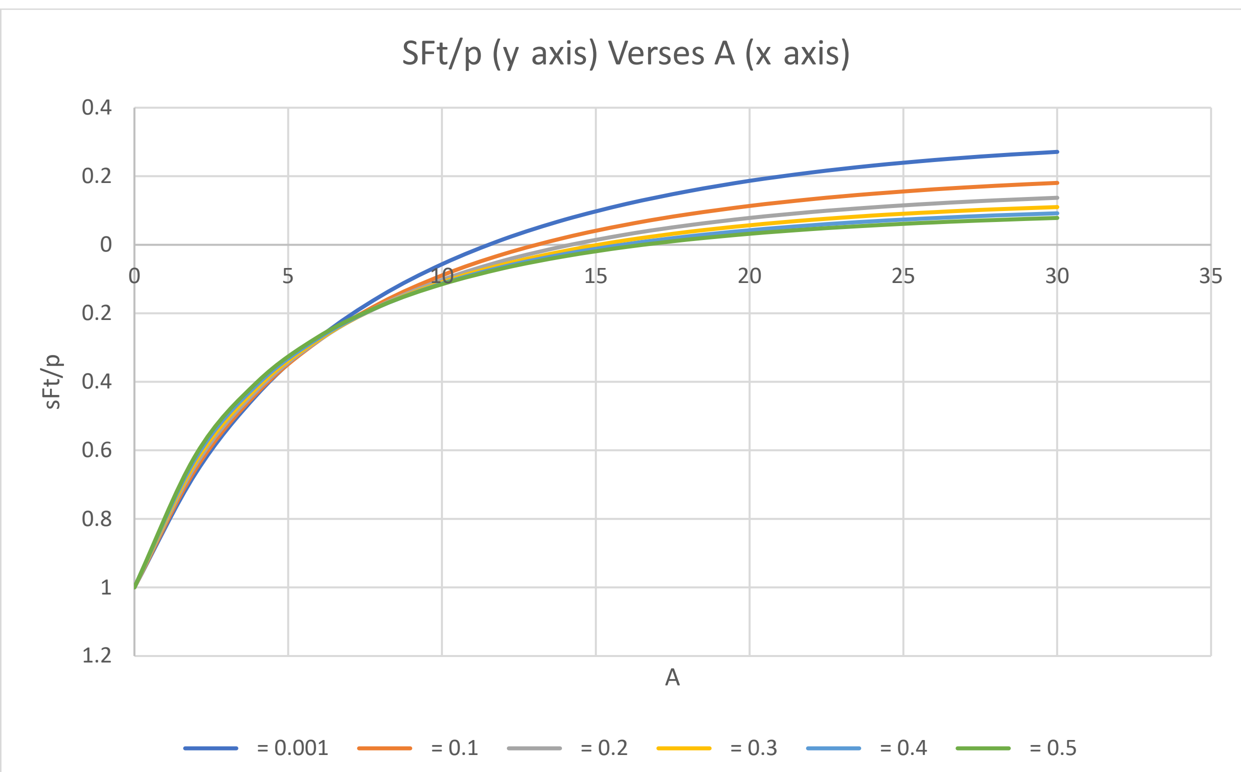
t/b	Amax	sft/p max	Rm	Rv
0.001	47.2	0.303748	0.108182	0.980359
0.1	47.85	0.206469	0.092543	0.602219
0.2	48.6	0.159955	0.082332	0.467557
0.3	49.35	0.131517	0.074818	0.390102
0.4	50.05	0.111985	0.068959	0.338085
0.5	50.85	0.097733	0.064001	0.300147

For the condition where *sft/p* is zero for soils see table below.

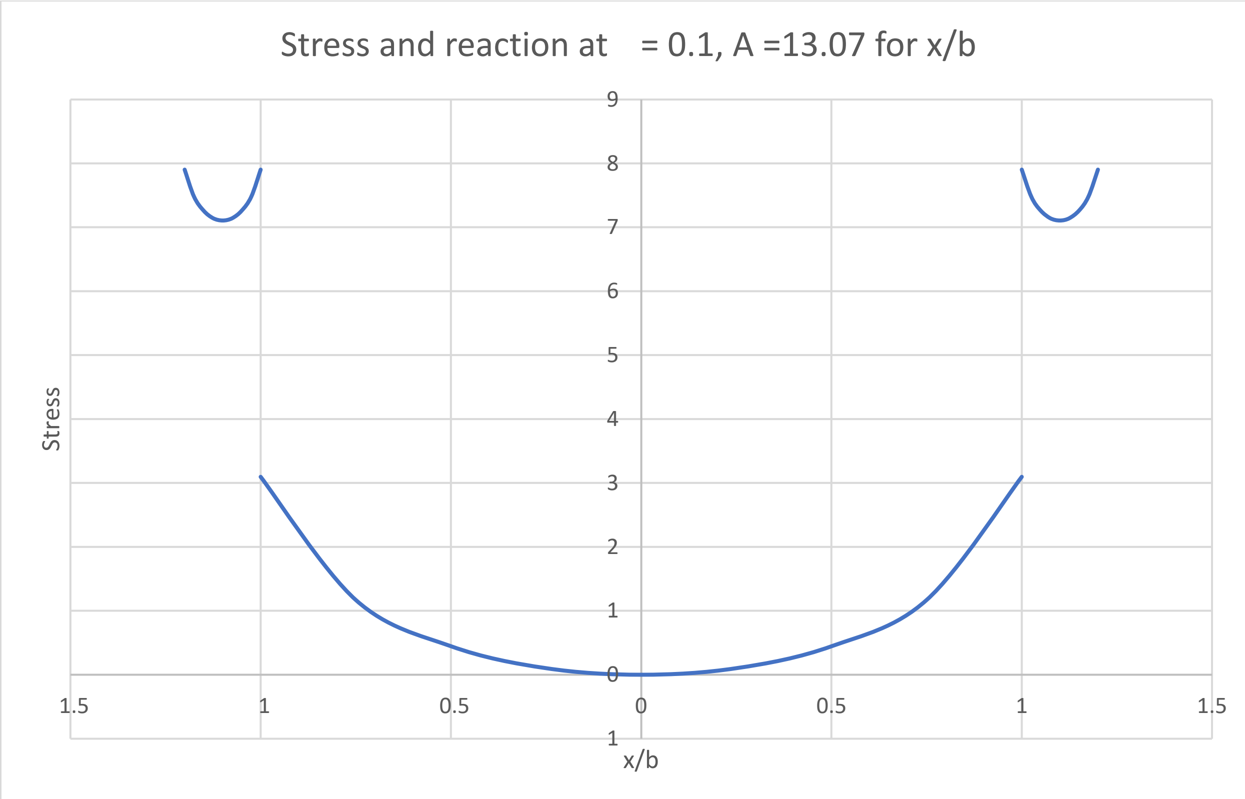
t/b	Amax	Stress	Rm	Rv
0.001	11.515	6.21E 05	0.399488	0.986743
0.1	13.07	4.6E 05	0.324267	0.733052
0.2	14.18	5.6E 05	0.279084	0.614458
0.3	15.08	9.59E 07	0.247479	0.537196
0.4	15.85	6.26E 05	0.223566	0.480777
0.5	16.53	9.06E 05	0.20456	0.436883

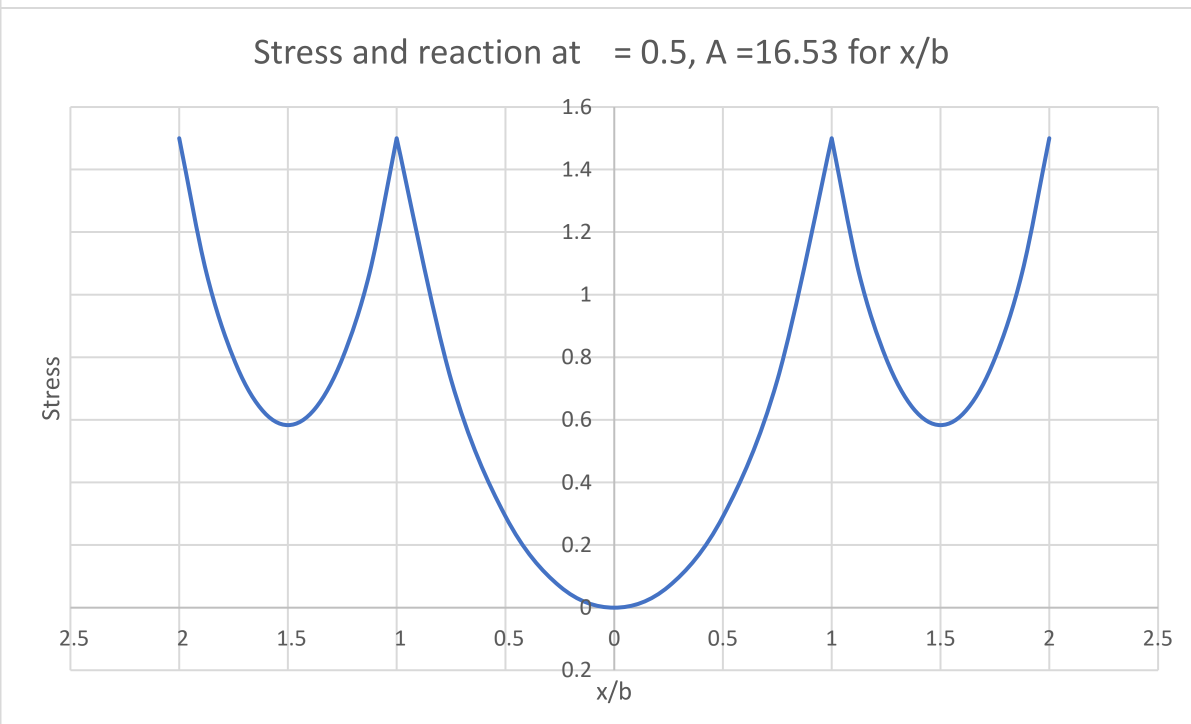
sFt/p Table 3 for different t/b

A	= 0.001	= 0.1	= 0.2	= 0.3	= 0.4	= 0.5
0	1	1	1	1	1	1
2	0.665	0.65291	0.64134	0.63083	0.62123	0.61234
4	0.43619	0.43123	0.42323	0.41506	0.40722	0.39978
6	0.27194	0.27935	0.27852	0.27542	0.27154	0.26739
8	0.14978	0.1702	0.17685	0.17896	0.1791	0.17825
10	0.05652	0.08904	0.10249	0.10929	0.11299	0.11501
12	0.016105	0.02714	0.04651	0.05732	0.06403	0.06845
14	0.073516	0.020962	0.00343	0.01761	0.02683	0.03324
16	0.119423	0.058891	0.030265	0.013281	0.001987	0.00606
18	0.156446	0.089122	0.056953	0.037639	0.024634	0.015253
20	0.186489	0.113411	0.078286	0.057045	0.042633	0.032159
22	0.210965	0.133033	0.095453	0.072622	0.057054	0.04569
24	0.230946	0.148938	0.109327	0.085189	0.068676	0.056587
26	0.247257	0.161844	0.120564	0.095357	0.078074	0.0654
28	0.260544	0.172309	0.129665	0.103591	0.085685	0.072541
30	0.271321	0.180764	0.137019	0.110248	0.091845	0.078328



In the following graph the pressure on the lagging is shown positive to see how the distribution is for various *x/b* and the reaction is shown.





We seek to find the deflection when starting from $n = 1$ we have

$$v_n = -\frac{\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^5} + \frac{2C2}{t}\frac{\sin\alpha_n b}{\alpha_n^3}}{EI + \frac{s}{2\beta\,\alpha_n^3}}$$

$$\sum_{n=1}^{\infty} v_n\, cos\,\alpha_n\, x = -\sum_{n=1}^{\infty}\frac{\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^5} + \frac{2C2}{t}\frac{\sin\alpha_n b}{\alpha_n^3}}{EI + \frac{s}{2\beta\,\alpha_n^3}}\cos\alpha_n x$$

$$\frac{EI\,v(x)}{\frac{5pb^4}{24}} = -\frac{EI}{\frac{5pb^4}{24}}\sum_{n=1}^{\infty}\frac{\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^5} + \frac{2C2}{t}\frac{\sin\alpha_n b}{\alpha_n^3}}{EI + \frac{s}{2\beta\,\alpha_n^3}}\cos\alpha_n x$$

$$\frac{EI\,v(x)}{\frac{5pb^4}{24}} = -\frac{1}{\frac{5pb}{24}}\sum_{n=1}^{\infty}\frac{\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^2} + \frac{2C2}{t}\sin\alpha_n b}{b^3\alpha_n^3 + \frac{sb^3}{2\beta EI}}\cos\alpha_n x$$

$$\frac{EI\,v(x)}{\frac{5pb^4}{24}} = -\frac{1}{\frac{5}{24}}\sum_{n=1}^{\infty}\frac{\frac{2}{tb}\frac{\sin\alpha_n b}{\alpha_n^2} + \frac{2C2}{tbp}\sin\alpha_n b}{b^3\alpha_n^3 + A}\cos\alpha_n x$$

$$\frac{EI\,v(x)}{\frac{5pb^4}{24}} = -\frac{1}{\frac{5\Gamma}{24}}\sum_{n=1}^{\infty}\frac{2(1+\Gamma)^2\frac{\sin\alpha_n b}{(\pi n)^2} + \frac{C2}{0.5pb^2}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3} + A}\cos\alpha_n x \;\ldots\ldots\ldots..75$$

Also, the deflection should go more negative to account for the deflection of the column which is PL/AE. Where P is the reaction equal to -2*pb* – 2*pt*. Also, we have the requirement to have the relative deflection to be zero at $x = \pm\, b$ to calculate the constant of integration C4. Rewrite the equations with these criteria: We have the deflection:

$$\frac{EI\,v(x)}{-\frac{5pb^4}{24}} = \frac{1}{\frac{5\Gamma}{24}}\sum_{n=1}^{\infty}\frac{2\frac{(1+\Gamma)^2}{(\pi n)^2} + \frac{C2}{0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + A}(\cos\alpha_n x - \cos\alpha_n b)\sin\alpha_n b \;\ldots\ldots\ldots\ldots\ldots 76$$

To verify the constant coefficient of the Fourier series we have:

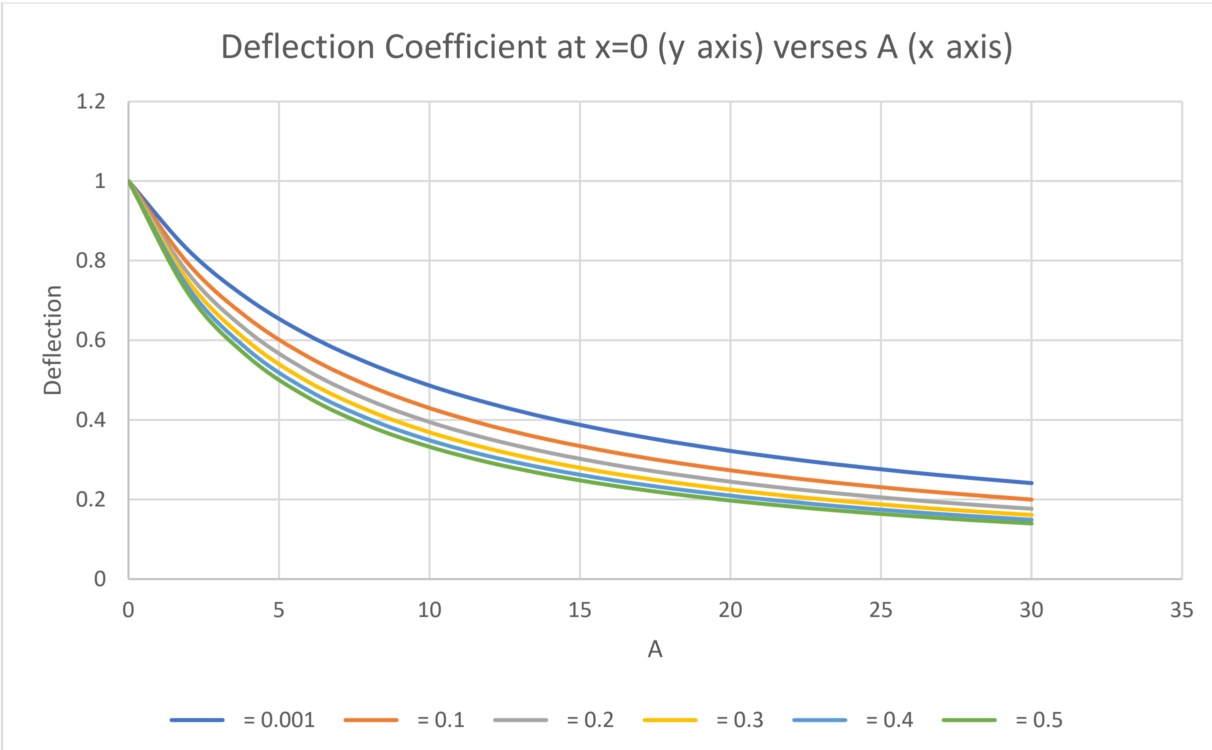
$$\frac{v_0}{2} = \frac{1}{b+t}\int_0^{b+t} v(x)\,dx = -\frac{5pb^4}{24EI}\frac{1}{b+t}\sum_{n=1}^{\infty}\frac{2\frac{(1+\Gamma)^2}{(\pi n)^2}+\frac{C2}{0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3}+A}(-\,(b+t)\cos\alpha_n b)\sin\alpha_n b$$

$$= -\frac{5pb^4}{24EI}\sum_{n=1}^{\infty}\frac{2\frac{(1+\Gamma)^2}{(\pi n)^2}+\frac{C2}{0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3}+A}(-\,\cos\alpha_n b)\sin\alpha_n b$$

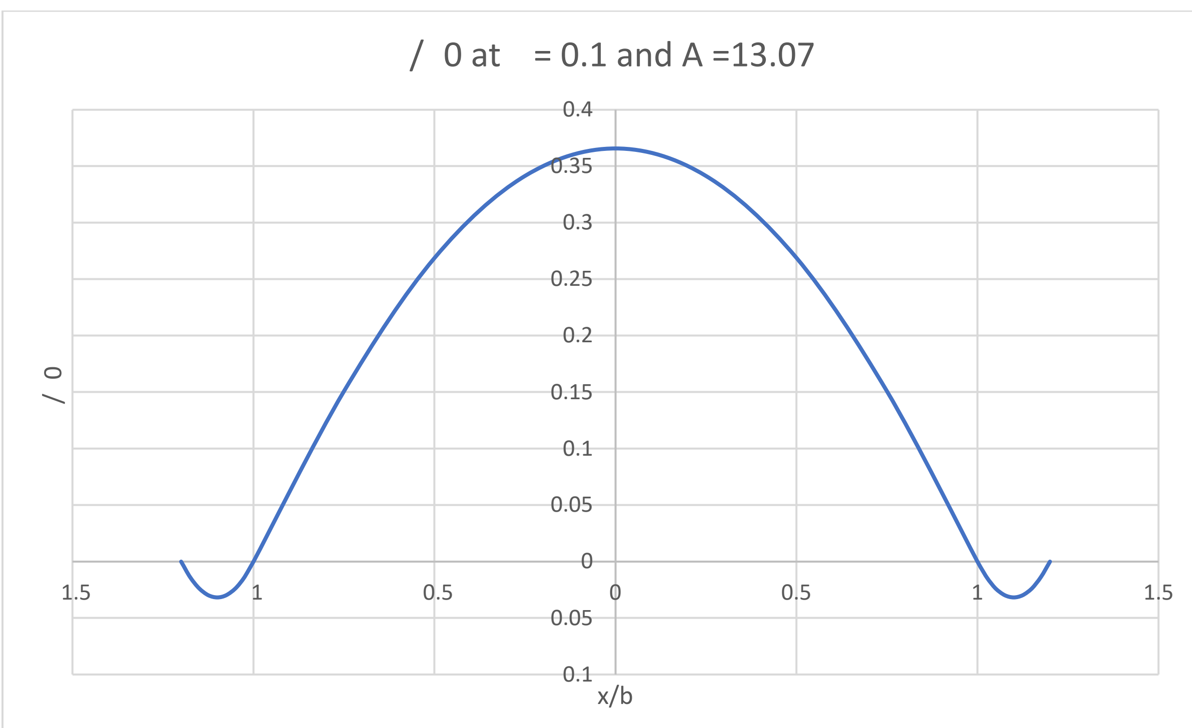
And it is verified and is defined not infinite.

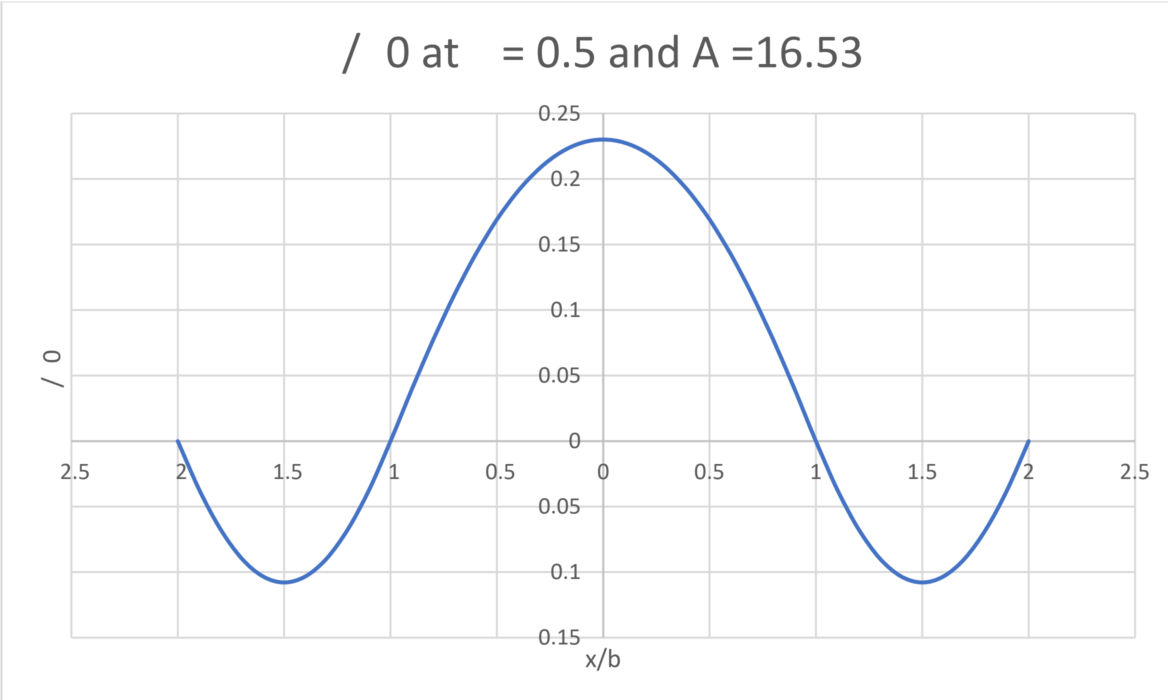
Deflection coefficient Table 4 for different A at x=0

A	= 0.001	= 0.1	= 0.2	= 0.3	= 0.4	= 0.5
0	1	1	1	1	1	1
2	0.8252038	0.7908272	0.7662538	0.7467072	0.7302261	0.715774
4	0.7025856	0.653851	0.6207229	0.5952991	0.5744631	0.556631
6	0.6118157	0.5572023	0.5214101	0.4946146	0.4730689	0.454923
8	0.5419117	0.4853627	0.449325	0.4228341	0.4018235	0.384331
10	0.4864201	0.4298704	0.3946272	0.3690826	0.349032	0.332484
12	0.4413	0.3857187	0.351708	0.3273322	0.308355	0.2928
14	0.4038919	0.3497559	0.3171366	0.2939724	0.2760578	0.261457
16	0.372374	0.3198991	0.2886964	0.2667087	0.2497977	0.236081
18	0.3454565	0.2947164	0.2648917	0.244013	0.2280303	0.21512
20	0.3222005	0.2731911	0.2446764	0.2248286	0.2096965	0.197518
22	0.3019064	0.2545813	0.2272973	0.2084011	0.1940457	0.18253
24	0.284042	0.2383332	0.2121978	0.1941777	0.180531	0.169616
26	0.2681955	0.2240246	0.1989581	0.181744	0.1687446	0.158375
28	0.2540431	0.2113284	0.1872555	0.1707833	0.1583762	0.148504
30	0.241327	0.1999871	0.1768377	0.1610495	0.1491856	0.139768



In the following graph the deflection coefficient on the lagging is shown positive at the lagging to see how the distribution is for various *x/b*. We see the deflection at the column is practically zero for *t*/*b* = 0.1.





Note: in the above chart the deflection at the column has a much larger moment of inertia so when multiplying by $-\frac{5pb^4}{24EI}$ the deflection is practically zero and match the previous article the compatibility of rotation is not required. The deflection at the center of lagging is shown at $x = 0$. However, if we define zero deflection at $x = \pm\,(b + t)$ instead of $\pm b$ and offset the chart, we see that the corners of the column deflect downward because of bearing. Allowing deflection at the column theoretically gives less arching since the more we have a ridged column, as zero deflection, the load will go to the stiffer column and reduce the load on the lagging or more arching. So, the presented solution in this article is more realistic. However, the deflection does not take into account the moment of inertia of the elastic media, so it is conservative. But for a media such as soils where tension is not allowed it is accurate and useful.

Finally, the rotation equation is:

$$\frac{EI\;v'(x)}{\frac{pb^3}{3}} = \frac{3}{\Gamma}\sum_{n=1}^{\infty} \frac{2(1+\Gamma)\dfrac{\sin\alpha_n b}{\pi n} + \dfrac{C2}{0.5pb^2}\dfrac{\pi n}{(1+\Gamma)}\sin\alpha_n b}{\dfrac{(\pi n)^3}{(1+\Gamma)^3} + A} \sin\alpha_n x \;\ldots\ldots..77$$

Finn [3] shows that the presence of friction between soil and wall requires a minor change in the stress function, due to $\varepsilon_x = 0$, which leads to

$$\sigma_y = \int_0^{\infty} -D\left[\frac{2\beta}{\beta+\rho} + \alpha y\right] e^{-\alpha y}\;\cos\alpha\, x\; d\alpha \ldots\ldots\ldots\ldots\ldots\ldots.\ldots\ldots.78$$

Solving for D shows that the solution has the same form as in the frictionless case but $1/(2\beta)$ is replaced by $2\beta/[(3\beta - \rho)(\beta + \rho)]$. Thus, may be used, if A is calculated as

$$A = \frac{4\beta^2}{(3\beta - \rho)(\beta + \rho)}\left[\frac{sb^3}{2\beta EI}\right] \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots 79$$

The conditions here are slightly different from those addressed by Finn [3] since ε_x is not quite zero. Thus, is not strictly valid, however it acts as an upper bound on A, and so is useful. We can consider the lagging or the beam to be glued to the mass media such that tension will act upward on the beam in the plain stress problem. Thus, in plain stress we have:

For a rough surface we have:

$$A = \frac{4}{(3 - \nu)(1 + \nu)}\left[\frac{sb^3 E_s}{2EI}\right] \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. 80$$

For a smooth surface we have:

$$A = \frac{sb^3 E_s}{2EI} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.. 81$$

In plain strain we have:

For a rough surface we have:

$$A = \frac{4(1 - \nu)(1 - \nu^2)}{(3 - 4\nu)(1 + \nu)}\left[\frac{sb^3 E_s}{2(1 - \nu^2)EI}\right] \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.. 82$$

For a smooth surface we have:

$$A = \frac{sb^3 E_s}{2(1 - \nu^2)EI} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots 83$$

Once D is found the stresses at $q = 0$ and $h = \infty$ are:

$$\sigma_y = -\sum_{n=\varphi}^{\infty} D\cos\alpha_n x\left(1 + \frac{\pi n}{(1 + \Gamma)}\frac{y}{b}\right)e^{-\alpha_n y} - p \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots 84$$

$$D = -\frac{s\alpha_n v_n}{\beta d_n} = -\frac{\alpha_n p s b^4}{2\beta EI\Gamma}\left[\frac{2(1 + \Gamma)^2 \dfrac{\sin\alpha_n b}{(\pi n)^2} + \dfrac{C2}{0.5pb^2}\sin\alpha_n b}{\dfrac{(\pi n)^3}{(1 + \Gamma)^3} + A}\right]$$

$$D=-\frac{s\alpha_n v_n}{\beta d_n}=p\frac{sb^3}{\beta EId_n\Gamma}\frac{\pi n}{1+\Gamma}\left[\frac{2(1+\Gamma)^2\frac{\sin\alpha_n b}{(\pi n)^2}+\frac{C2}{0.5pb^2}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3}+A}\right]$$

$$d_n=-2$$

$$D=-p\frac{A}{\Gamma}\left[\frac{2\frac{(1+\Gamma)}{\pi n}+\frac{\pi n}{(1+\Gamma)}\frac{C2}{0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3}+A}\right]\sin\alpha_n b \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.85$$

$$\sigma_x=k_0\sum_{n=\varphi}^{\infty}D\cos\alpha_n x\left(1+\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y}-2k_0\sum_{n=\varphi}^{\infty}De^{-\alpha_n y}\cos\alpha_n x-k_0 p\ldots\ldots\ldots\ldots..86$$

$$\tau_{xy}=-\sum_{n=\varphi}^{\infty}\alpha_n Dye^{-\alpha_n y}\sin\alpha_n x$$

$$\tau_{xy}=-\sum_{n=\varphi}^{\infty}\frac{\pi n}{1+\Gamma}D\frac{y}{b}e^{-\alpha_n y}\sin\alpha_n x\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.87$$

Example 1:

We have a 4 ft x 4 ft opening to be install in a dam powerhouse for a mechanical duct. The wall is concrete 30 ft high and 3 ft thick. We can assume that qualify for an infinite height and can use the above solution with plain stress. Note at $y = 0$ we have $\sigma_y = \sigma_x/k_0$

		b=	2 ft		4 ft opening
		t =	1 ft	s =	3 ft
	Concrete	150 pcf	at the powerhouse at the dam		
p=	13500	lbs/ft	30 ft height x 3 ft thick		
p=	1125	lbs/in	Pick t/b = 0.5		
p/ inch =	1125	psi	v=	0.2	
Ft=	87.63561	psi	We use an old code Ft = 1.6	3000	
sFt/p/ =	0.389492	> 0.09773	F'c =	3000 psi	
Use sft/p max			Allowable Shear = 0.02f'c =	60	

For a rough surface we have:

$$A = \frac{4}{(3-\nu)(1+\nu)}\left[\frac{sb^3 E_s}{2EI}\right]$$

Es = E

	Thus	IA =	14.28571		
	A =	50.85	for sFt/p = 0.0977333		
	thus I =	0.2809383	ft^4		
	thus I =	5825.5373	in^4 =	sh^3/12	
	h^3 =	1941.8458	h =	12.47589	in
	Sx =	933.8872	in^3 A =	449.1321	in^2
Mu =	1.3pb^2/2 =	35100	Lbs ft	Ultimate Moment	
Mu reduced =		2965.827	k ft at x = 0.615b	Rm =	0.084496
	ft = M/Sx	38.10944	< Ft =	87.63561	psi
	Rv =	0.025824	at x/b = 0.3787	Max shear	
	1.3fv =	3.03	< Fv =	60	psi
	Ec = Es = E = 57000 3000 =			3122019	psi
	0 = 5*p*b^4/24EI =		0.00428	inch	
	/ 0 =	0.0861896	Thus =	0.00037	inch
Thus, the relative deflection at the center is =				0.00037	inch

For a smooth surface we have:

$$A = \frac{sb^3 E_s}{2EI}$$

Es = E

	Thus	IA =	12		
	A =	50.85	for sFt/p = 0.0977333		
	thus I =	0.2359882	ft^4		
	thus I =	4893.4513	in^4 =	sh^3/12	
	h^3 =	1631.1504	h =	11.7715	in
	Sx =	831.4075	in^3 A =	423.774	in^2
Mu =	1.3pb^2/2 =	35100	Lbs ft	Ultimate Moment	
Mu reduced =		2965.827	k ft at x=0.615b	Rm =	0.084496
	ft = M/Sx	42.807	< Ft =	87.63561	psi
	Rv =	0.025824	at x/b = 0.3787	Max shear	
	1.3fv =	3.21	< Fv =	60	psi
	Ec = Es = E = 57000 3000 =			3122019	psi
	0 = 5*p*b^4/24EI =		0.00509	inch	
	/ 0 =	0.0861896	Thus =	0.000439	inch
Thus, the relative deflection at the center is =				0.000439	inch

The deflection does not take into account there is 30 ft above with high moment of inertia. So, this deflection is not actual.

Now, *Rm* and *Rv* was found for A = 50.85 from the tables. So, we can see that the opening can be installed. The infinite height used as an assumption should be reduced by 12.476 inch which does not change the solution.

Example 1A:

A 15 ft cut permanent shoring retaining soils and it is desirable to use recycled plastic for lagging. Reference: https://american-plasticlumber.com/wp-content/uploads/2016/03/apl_web_str_ti_hdpe_tech_data.pdf

The piles are 8 ft O.C. and the flange is 6-inch-wide on the steel pile. Thus, we have:

$$A = \frac{sb^3E_s}{2EI}$$

Smooth surface

b=	45	in		8 ft spacing	6 inch flange
t =	3	in	s =	3.50	in 4x4 plastic lagging
Ep =	221260	psi			
=	0.3				
1/ß=Es/(1 ^2) =	1221	psi Es = 1111 psi			
I =	12.51	in^4			
A =	70.37				
=	0.067				
For =0.067 Amax =	12.62	sFt/p =		3.97E 05	

Thus, cannot use A = 70.37 it produces tension in soil

The allowable reduction is:	Rm =		0.344581	at x/b = 0
Rv =		0.573092	at x/b =	0.922222
15 ft Shoring Ka = 0.5, = 120 pcf p = 0.9 ksf				
p at s = 3.5 in or 0.2917 ft=			0.2625	ksf per s
Vmax =		0.564138	kip	
Mmax =		0.635994	ft kip	
Sx =		7.145833	in^3	
M/Sx =		1068.024	psi <	2114 psi OK
Assume shear as wood				
Area of member =			12.25	in^2
= 1.5 Vmax / Area =			69.07806	psi < 95 psi OK

Thus, 4x4 Plastic Lagging Is OK

Closed form solution #2 fixed end span lagging with *h* = ∞:

The equations remain the same as the previous solution only the constant C2 is different. In the previous solution C2 could not be define at $x = b$ in a Fourier series because it was a step function and C2 was done carefully. In the fixed solution C2 can be found by setting the rotations to be zero at $x = \pm b$ and the rotation at that point is not a step and can be found by setting rotation equation 71 to be zero at $x = \pm b$. Thus:

$$0 = \frac{3}{\Gamma}\sum_{n=1}^{\infty}\frac{2(1+\Gamma)\frac{\sin\alpha_n b}{\pi n}+\frac{C2}{0.5pb^2}\frac{\pi n}{(1+\Gamma)}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3}+A}\sin\alpha_n b \quad \ldots\ldots 88$$

Or

$$-\sum_{n=1}^{\infty}\frac{2(1+\Gamma)\frac{(\sin\alpha_n b)^2}{\pi n}}{\frac{(\pi n)^3}{(1+\Gamma)^3}+A} = \frac{C2}{0.5pb^2}\sum_{n=1}^{\infty}\frac{\frac{\pi n}{(1+\Gamma)}}{\frac{(\pi n)^3}{(1+\Gamma)^3}+A}(\sin\alpha_n b)^2 \quad \ldots\ldots 89$$

$$\frac{C2}{0.5pb^2} = -\frac{\sum_{n=1}^{\infty}\frac{2(1+\Gamma)\frac{(\sin\alpha_n b)^2}{\pi n}}{\frac{(\pi n)^3}{(1+\Gamma)^3}+A}}{\sum_{n=1}^{\infty}\frac{\frac{\pi n}{(1+\Gamma)}}{\frac{(\pi n)^3}{(1+\Gamma)^3}+A}(\sin\alpha_n b)^2} \quad \ldots\ldots 90$$

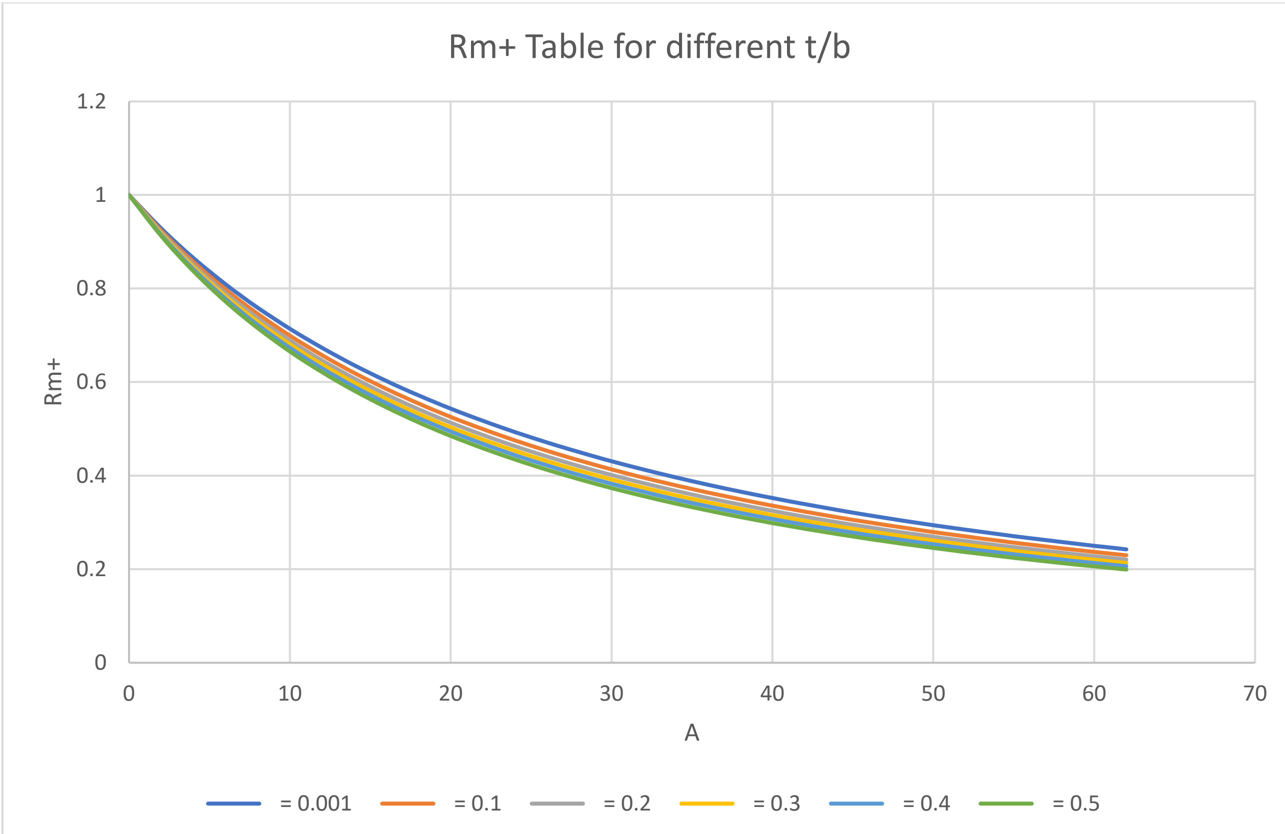
The positive moment at $x = 0$ is

Note: the maximum positive moment may not be at $x = 0$ depending on A

$$\frac{M+}{\frac{pb^2}{6}} = Rm+ = 3\left[\frac{1}{\Gamma}\sum_{n=1}^{\infty}\frac{4\sin\alpha_n b - 2A\left(\frac{C2}{0.5pb^2}\right)\frac{(1+\Gamma)}{\pi n}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3}+A}+\frac{C2}{0.5pb^2}\right] \quad \ldots\ldots 91$$

Rm+ Table 5 for different t/b

A	= 0.001	= 0.1	= 0.2	= 0.3	= 0.4	= 0.5
0	0.99999	0.999988	0.999985	0.999983	0.99998	0.999977
2	0.928426	0.92366	0.92021	0.917441	0.91492	0.912341
4	0.865258	0.856911	0.850908	0.846102	0.841721	0.837238
6	0.809117	0.798075	0.790179	0.783867	0.778106	0.772207
8	0.758917	0.745851	0.736554	0.72913	0.722343	0.71539
10	0.713784	0.699208	0.68888	0.680639	0.673093	0.665358
12	0.673006	0.657316	0.646241	0.637407	0.629304	0.620994
14	0.635999	0.619503	0.607899	0.598643	0.590137	0.581412
16	0.602278	0.585218	0.573251	0.563705	0.554918	0.545901
18	0.571437	0.554002	0.541804	0.532071	0.523095	0.513883
20	0.543133	0.525474	0.513147	0.503306	0.494215	0.484884
22	0.517077	0.499312	0.486936	0.47705	0.467901	0.458511
24	0.493021	0.475244	0.462881	0.452999	0.443837	0.434435
26	0.470752	0.453037	0.440737	0.430897	0.421758	0.412382
28	0.450086	0.432492	0.420292	0.410524	0.401437	0.392116
30	0.430862	0.413437	0.401367	0.391694	0.382681	0.373439
32	0.412942	0.395721	0.383804	0.374245	0.365323	0.356179
34	0.396203	0.379214	0.367469	0.358037	0.34922	0.340188
36	0.380537	0.363803	0.352242	0.342948	0.334246	0.325338
38	0.36585	0.349387	0.33802	0.328871	0.320293	0.311518
40	0.352058	0.335877	0.324711	0.315713	0.307265	0.298628
42	0.339084	0.323195	0.312234	0.303391	0.295077	0.286584
44	0.326863	0.311271	0.300518	0.291832	0.283656	0.27531
46	0.315334	0.300042	0.289498	0.280972	0.272935	0.264737
48	0.304443	0.289454	0.279119	0.270751	0.262854	0.254807
50	0.294142	0.279455	0.269328	0.261119	0.253363	0.245466
52	0.284387	0.27	0.260081	0.252029	0.244413	0.236666
54	0.275139	0.26105	0.251335	0.243439	0.235963	0.228365
56	0.266361	0.252567	0.243053	0.235312	0.227974	0.220524
58	0.258021	0.244518	0.235203	0.227613	0.220412	0.213108
60	0.250089	0.236873	0.227752	0.220312	0.213246	0.206086
62	0.242538	0.229603	0.220674	0.21338	0.206447	0.19943



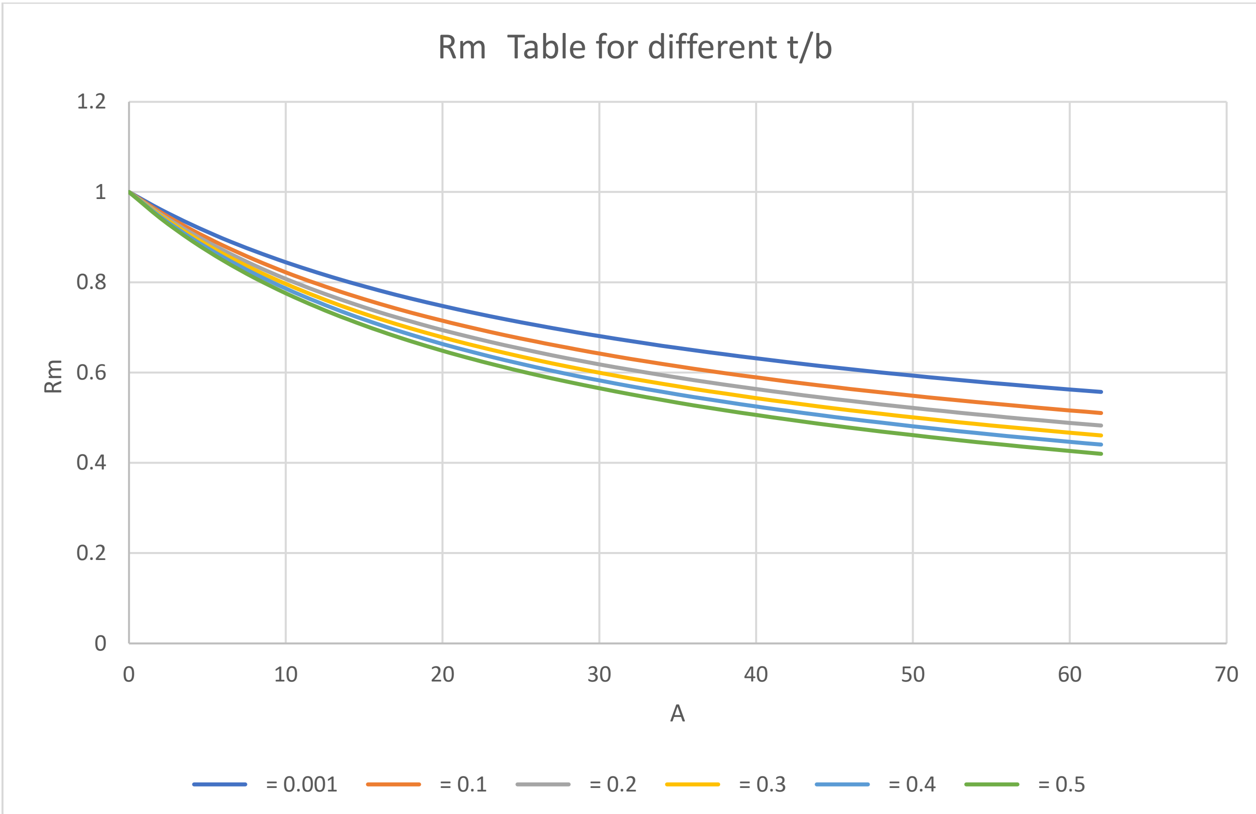
Rm+ Table for different t/b
1.2
1
0.8
0.6
0.4
0.2
0
Rm+
0
10
20
30
40
50
60
70
A
= 0.001
= 0.1
= 0.2
= 0.3
= 0.4
= 0.5

The negative moment at $x = \pm b$ is

$$\frac{M-}{\frac{-pb^2}{3}} = Rm-= -\frac{3}{2}\left[\frac{1}{\Gamma}\sum_{n=1}^{\infty}\frac{4\sin\alpha_n b - 2A\left(\frac{C2}{0.5pb^2}\right)\frac{(1+\Gamma)}{\pi n}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3}+A}\cos\alpha_n b + \frac{C2}{0.5pb^2}\right] \quad .92$$

Rm- Table 6 for different t/b

A	= 0.001	= 0.1	= 0.2	= 0.3	= 0.4	= 0.5
0	1.000005	1.000006	1.000007	1.000009	1.00001	1.000011
2	0.96165	0.955499	0.951356	0.948115	0.945185	0.942193
4	0.927536	0.91628	0.90877	0.902914	0.897613	0.892197
6	0.896979	0.881437	0.871157	0.863159	0.85591	0.848502
8	0.869433	0.850255	0.837671	0.827898	0.819026	0.809958
10	0.844461	0.822169	0.807649	0.796387	0.786148	0.77568
12	0.821704	0.796725	0.780562	0.768039	0.756635	0.744975
14	0.80087	0.773553	0.755987	0.742385	0.729978	0.717294
16	0.781714	0.752351	0.733575	0.719045	0.705767	0.692195
18	0.764034	0.732866	0.713042	0.697705	0.683665	0.66932
20	0.747656	0.71489	0.694152	0.678108	0.663398	0.648373
22	0.732435	0.698245	0.676705	0.66004	0.644734	0.62911
24	0.718246	0.68278	0.660534	0.643318	0.627481	0.611327
26	0.704982	0.668369	0.645497	0.627791	0.611478	0.59485
28	0.69255	0.654901	0.631472	0.613328	0.596585	0.579535
30	0.680869	0.642281	0.618354	0.599816	0.582684	0.565255
32	0.66987	0.630427	0.606054	0.587159	0.569673	0.551904
34	0.659489	0.619265	0.594491	0.575274	0.557464	0.539388
36	0.649672	0.608734	0.583598	0.564086	0.545981	0.527627
38	0.640373	0.598777	0.573313	0.553532	0.535155	0.516551
40	0.631547	0.589346	0.563584	0.543557	0.524929	0.506097
42	0.623156	0.580396	0.554363	0.534109	0.51525	0.496212
44	0.615167	0.57189	0.545608	0.525145	0.506071	0.486846
46	0.607549	0.563791	0.537283	0.516626	0.497353	0.477958
48	0.600275	0.556069	0.529354	0.508517	0.489058	0.469508
50	0.593319	0.548697	0.52179	0.500786	0.481155	0.461464
52	0.58666	0.541648	0.514566	0.493406	0.473613	0.453794
54	0.580276	0.5349	0.507656	0.48635	0.466406	0.446471
56	0.574151	0.528433	0.501038	0.479596	0.45951	0.439471
58	0.568266	0.522227	0.494694	0.473123	0.452904	0.43277
60	0.562607	0.516266	0.488604	0.466912	0.446569	0.426348
62	0.557159	0.510534	0.482752	0.460947	0.440486	0.420187

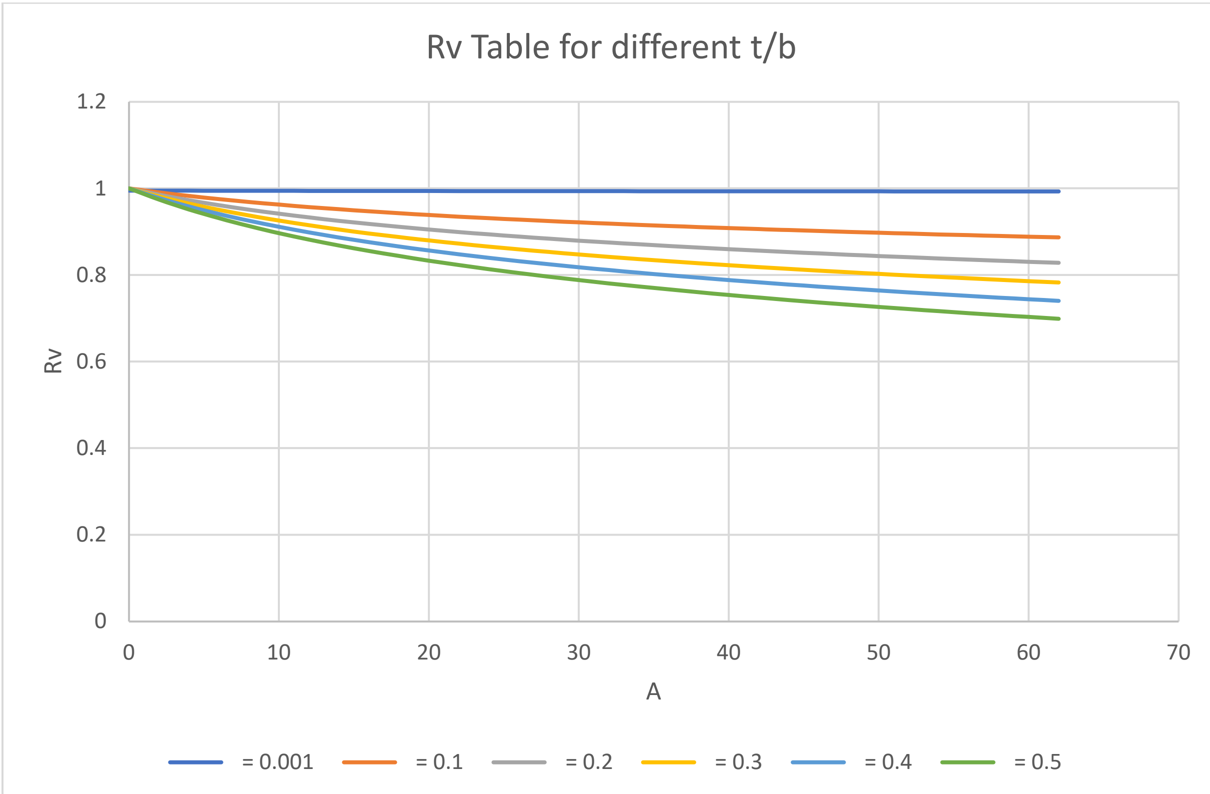

Rm Table for different t/b
Rm
A
1.2
1
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
60
70
= 0.001
= 0.1
= 0.2
= 0.3
= 0.4
= 0.5

The shear at $x = \pm b^*$ is

$$Rv = \frac{1}{\Gamma}\sum_{n=1}^{\infty} \frac{2\frac{\pi n}{(1+\Gamma)} - A\frac{C2}{0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + A} \sin \alpha_n b \sin \alpha_n b^* \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 93$$

Rv Table 7 for different t/b b=b*

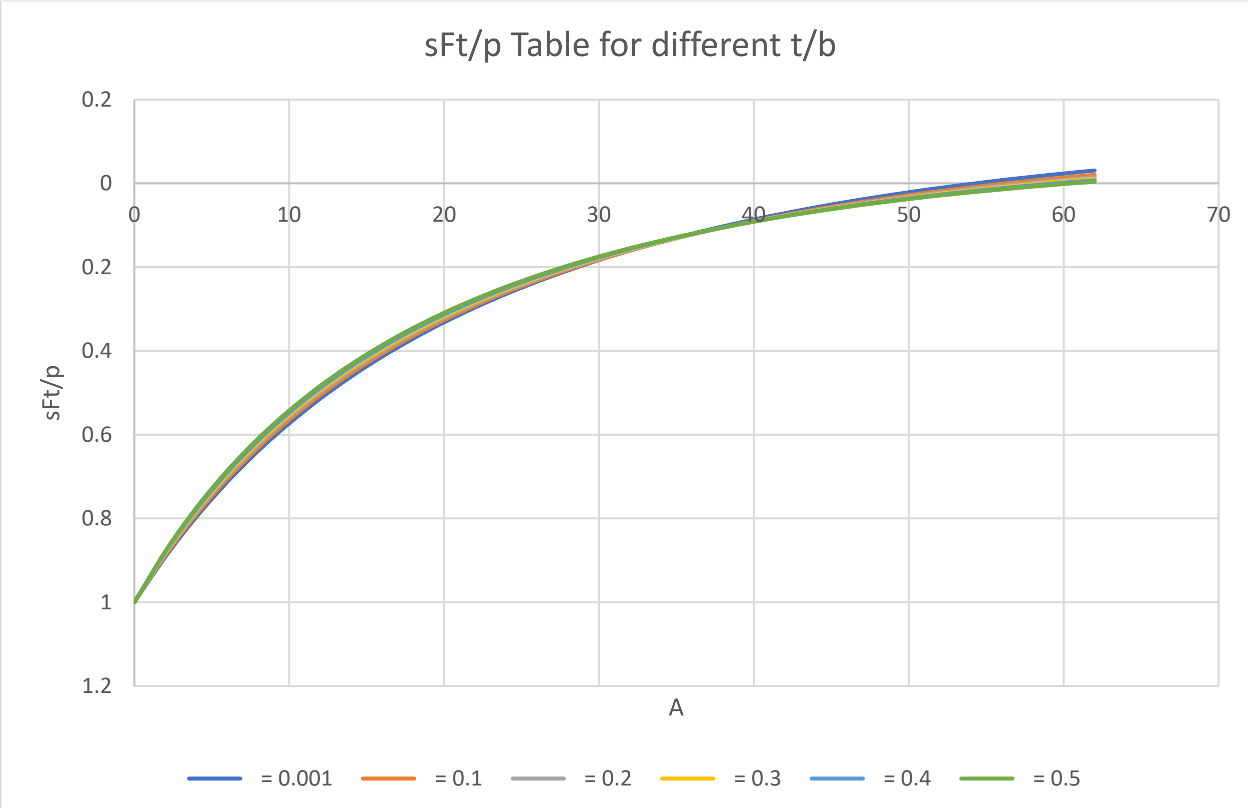
A	= 0.001	= 0.1	= 0.2	= 0.3	= 0.4	= 0.5
0	0.99493	0.999939	0.999964	0.999971	0.999975	0.999977
2	0.994785	0.990737	0.985426	0.981395	0.977764	0.974056
4	0.994654	0.982545	0.972578	0.965036	0.958237	0.951295
6	0.994535	0.975192	0.961116	0.950485	0.940887	0.93109
8	0.994425	0.968541	0.950808	0.937429	0.925329	0.912982
10	0.994325	0.962485	0.941468	0.925622	0.911266	0.896621
12	0.994232	0.956938	0.932953	0.914873	0.898462	0.881731
14	0.994145	0.95183	0.925142	0.905025	0.886731	0.868092
16	0.994064	0.947103	0.917941	0.895953	0.875922	0.855528
18	0.993987	0.94271	0.911271	0.887556	0.865912	0.843896
20	0.993916	0.93861	0.905065	0.879746	0.856598	0.833077
22	0.993848	0.93477	0.899269	0.872454	0.847897	0.822972
24	0.993783	0.931161	0.893836	0.86562	0.839738	0.8135
26	0.993722	0.927759	0.888726	0.859192	0.832059	0.804591
28	0.993664	0.924543	0.883907	0.853129	0.824812	0.796186
30	0.993608	0.921494	0.879348	0.847392	0.81795	0.788234
32	0.993555	0.918598	0.875024	0.84195	0.811437	0.78069
34	0.993503	0.915839	0.870913	0.836774	0.805241	0.773519
36	0.993454	0.913207	0.866997	0.831841	0.799331	0.766685
38	0.993407	0.91069	0.863258	0.82713	0.793684	0.76016
40	0.993361	0.908279	0.859682	0.822621	0.788277	0.753919
42	0.993317	0.905965	0.856255	0.818298	0.783091	0.74794
44	0.993274	0.903742	0.852965	0.814146	0.778109	0.742201
46	0.993233	0.901602	0.849802	0.810152	0.773315	0.736685
48	0.993193	0.899539	0.846756	0.806304	0.768695	0.731377
50	0.993154	0.897548	0.84382	0.802593	0.764237	0.726261
52	0.993116	0.895624	0.840985	0.799007	0.759931	0.721325
54	0.993079	0.893763	0.838245	0.79554	0.755765	0.716557
56	0.993043	0.89196	0.835593	0.792182	0.751731	0.711947
58	0.993008	0.890212	0.833025	0.788927	0.747821	0.707484
60	0.992974	0.888516	0.830533	0.78577	0.744028	0.70316
62	0.992941	0.886868	0.828115	0.782702	0.740343	0.698968


Rv Table for different t/b
1.2
1
0.8
Rv 0.6
0.4
0.2
0
0
10
20
30
40
50
60
70
A
= 0.001
= 0.1
= 0.2
= 0.3
= 0.4
= 0.5

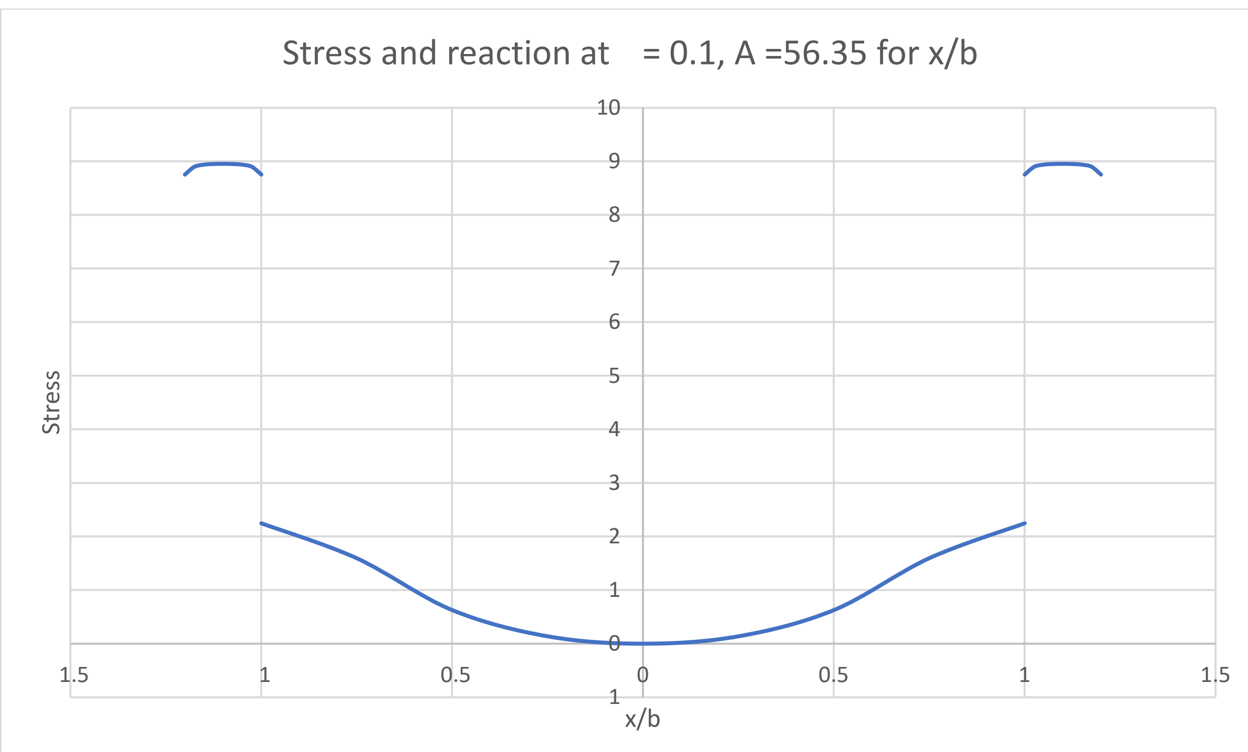
$$\frac{\sigma_y}{p}(at\ y=0\ and\ x=0)=\frac{sft}{p}=\frac{A}{\Gamma}\sum_{n=1}^{\infty}\frac{2\frac{(1+\Gamma)}{\pi n}+\frac{\pi n}{(1+\Gamma)}\frac{C2}{0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3}+A}\sin\alpha_n b-1\ldots 94$$

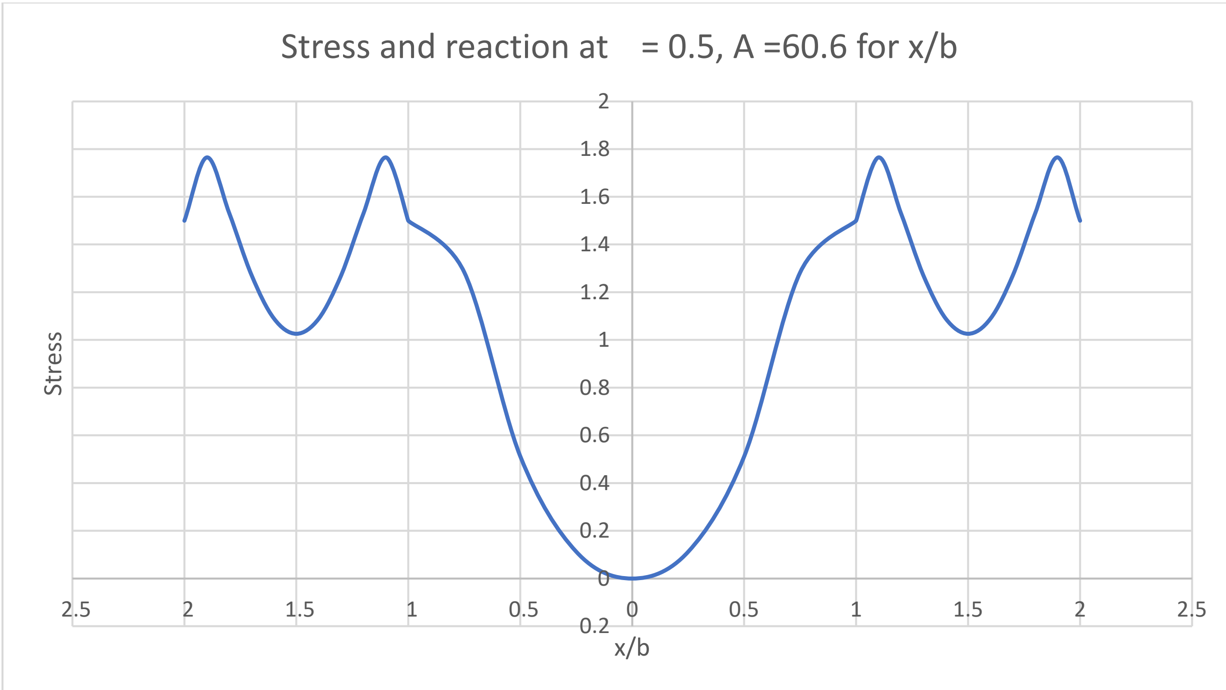
sFt/p Table 8 for different t/b

A	= 0.001	= 0.1	= 0.2	= 0.3	= 0.4	= 0.5
0	1	1	1	1	1	1
2	0.89135	0.88758	0.8847	0.88234	0.88017	0.87795
4	0.79633	0.79016	0.78545	0.78158	0.77801	0.77435
6	0.71269	0.70512	0.6993	0.69451	0.69009	0.68555
8	0.63866	0.63038	0.62399	0.61871	0.61382	0.60879
10	0.5728	0.56433	0.55773	0.55226	0.54718	0.54195
12	0.51394	0.50565	0.49912	0.49366	0.48859	0.48336
14	0.46113	0.45328	0.447	0.44173	0.4368	0.43171
16	0.41357	0.40636	0.40046	0.39547	0.39079	0.38595
18	0.37061	0.36415	0.35873	0.3541	0.34974	0.34522
20	0.33167	0.32606	0.32119	0.31697	0.31298	0.30883
22	0.29629	0.29159	0.28729	0.28352	0.27993	0.27619
24	0.26406	0.2603	0.25661	0.2533	0.25013	0.24681
26	0.23464	0.23182	0.22875	0.22592	0.22318	0.2203
28	0.20772	0.20586	0.2034	0.20104	0.19874	0.19631
30	0.18305	0.18213	0.18028	0.1784	0.17653	0.17454
32	0.1604	0.1604	0.15915	0.15774	0.1563	0.15474
34	0.13957	0.14048	0.13981	0.13885	0.13784	0.13671
36	0.12039	0.12217	0.12207	0.12156	0.12096	0.12024
38	0.1027	0.10533	0.10577	0.1057	0.1055	0.10518
40	0.08637	0.08981	0.09078	0.09113	0.09132	0.09139
42	0.07128	0.07551	0.07698	0.07772	0.07829	0.07874
44	0.05732	0.06229	0.06425	0.06538	0.06631	0.06713
46	0.04439	0.05008	0.0525	0.054	0.05528	0.05644
48	0.03241	0.03878	0.04165	0.04351	0.04511	0.04661
50	0.0213	0.02832	0.03161	0.03381	0.03574	0.03755
52	0.01098	0.01863	0.02232	0.02484	0.02708	0.02919
54	0.00141	0.00964	0.01372	0.01655	0.01908	0.02148
56	0.007489	0.00131	0.00574	0.00887	0.01168	0.01436
58	0.015757	0.006429	0.001649	0.00176	0.00484	0.00778
60	0.023445	0.013613	0.008506	0.004829	0.001499	0.00169
62	0.030595	0.020286	0.014869	0.010938	0.007365	0.003943



In the following graph the pressure on the lagging is shown positive to see how the distribution is for various *x/b* and the reaction is shown.





Thus *sft/p* has a maximum positive tension value see table below.

t/b	Amax	sft/p max	Rm+	Rm	Rv
0.001	140.5	0.114742	0.098186	0.432431	0.991999
0.1	141.25	0.098316	0.091545	0.379861	0.84313
0.2	141.25	0.088702	0.087597	0.350644	0.764456
0.3	141	0.081035	0.084239	0.32706	0.701593
0.4	140.5	0.073923	0.08117	0.305358	0.644224
0.5	141	0.067442	0.077476	0.28453	0.592369

For the condition where *sft/p* is zero for soils see table below.

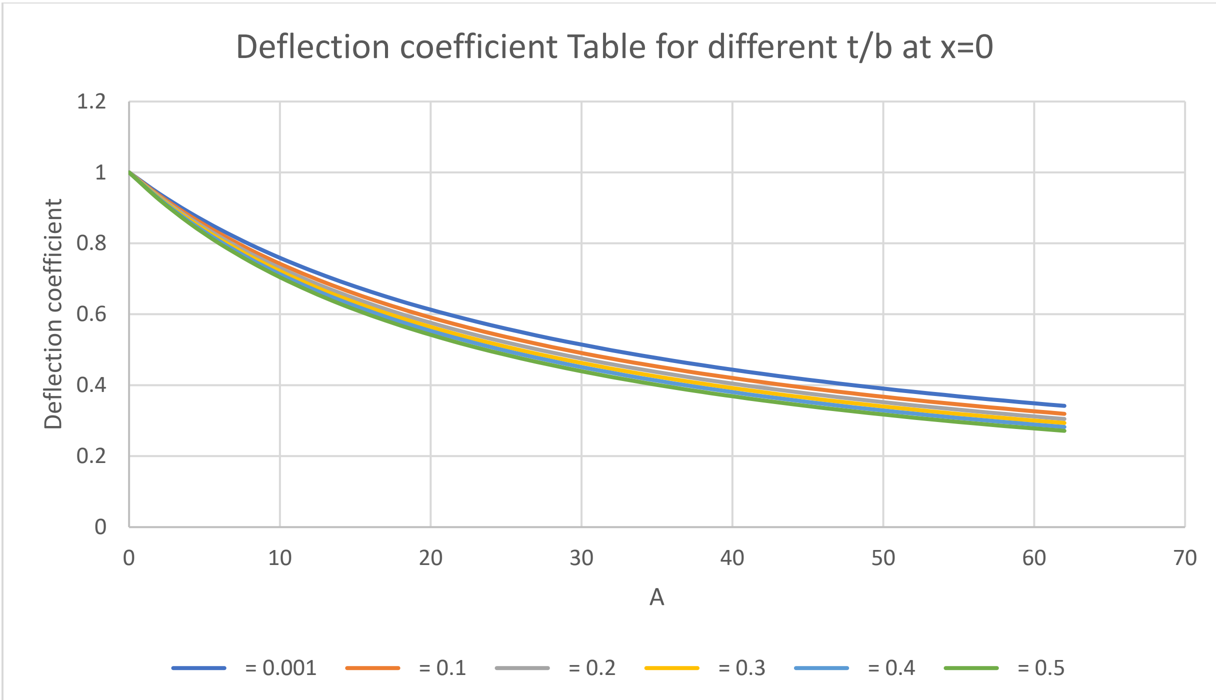
t/b	Amax	Stress	Rm+	Rm	Rv
0.001	54.3	3.1E 05	0.273793	0.579342	0.993073
0.1	56.35	8.8E 05	0.251128	0.527328	0.89165
0.2	57.55	3.35E 05	0.236933	0.496099	0.833596
0.3	58.5	6.5E 05	0.225752	0.471547	0.788129
0.4	59.5	3.9E 05	0.215002	0.448129	0.744966
0.5	60.6	4.77E 05	0.204052	0.424473	0.701889

Now the deflection

$$\frac{EI\ v(x)}{-\frac{pb^4}{24}} = \frac{1}{\frac{\Gamma}{24}} \sum_{n=1}^{\infty} \frac{2\frac{(1+\Gamma)^2}{(\pi n)^2} + \frac{C2}{0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + A} (\cos \alpha_n x - \cos \alpha_n b) \sin \alpha_n b \quad \dots\dots\dots\dots\dots 95$$

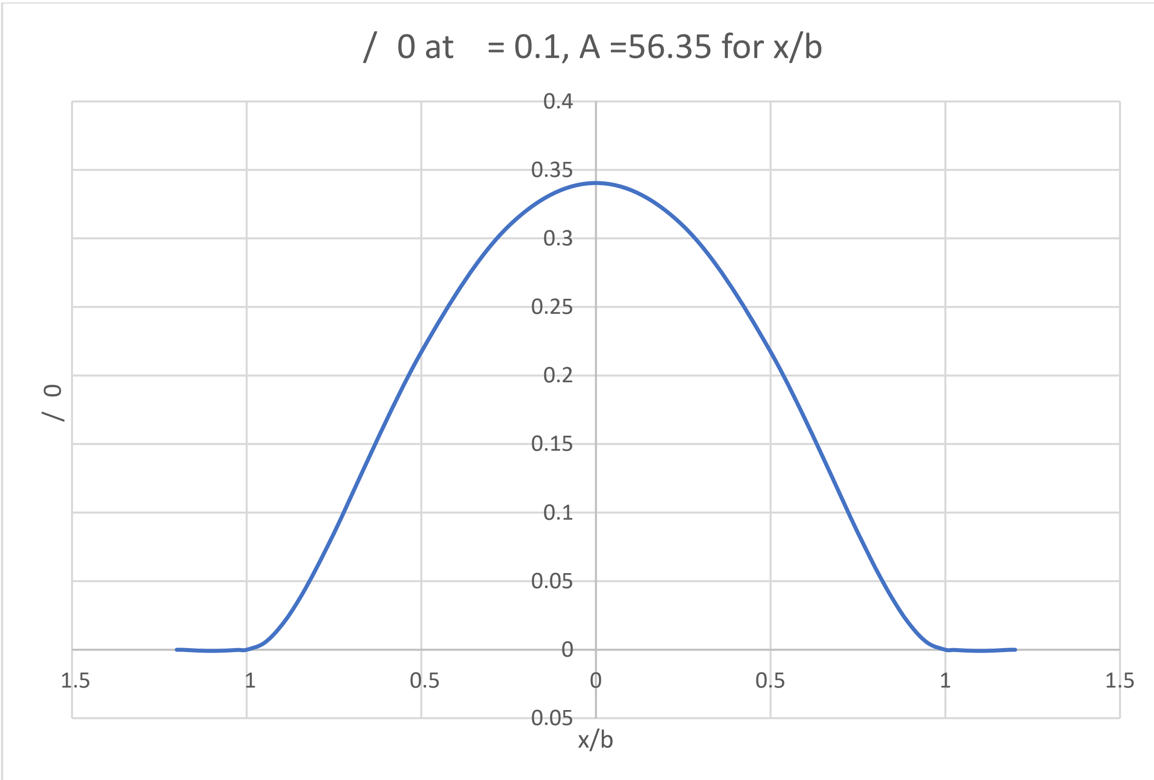
Deflection coefficient Table 9 for different x=0

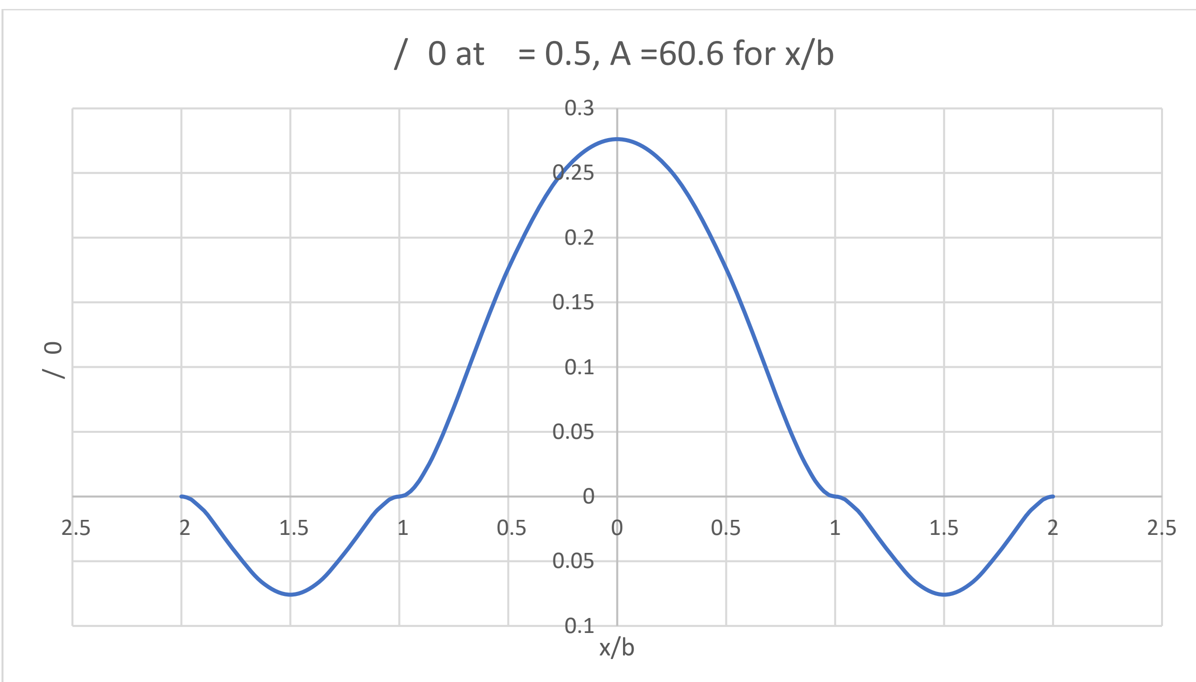
A	= 0.001	= 0.1	= 0.2	= 0.3	= 0.4	= 0.5
0	0.99998	0.999975	0.999971	0.999966	0.99996	0.999954
2	0.940169	0.93502	0.931359	0.928444	0.925795	0.923088
4	0.887196	0.878028	0.871562	0.866427	0.861758	0.856983
6	0.83995	0.82762	0.818985	0.812141	0.80591	0.799535
8	0.79755	0.782717	0.772395	0.764226	0.756778	0.749154
10	0.759287	0.742465	0.730825	0.721623	0.71322	0.704616
12	0.724582	0.706176	0.693505	0.683496	0.674342	0.664965
14	0.692961	0.673294	0.659816	0.649176	0.639428	0.629441
16	0.664031	0.643359	0.629253	0.618121	0.607904	0.597436
18	0.637461	0.615992	0.601401	0.589886	0.5793	0.568454
20	0.612974	0.590877	0.575913	0.564104	0.553229	0.542088
22	0.590334	0.567747	0.552503	0.54047	0.52937	0.518002
24	0.569339	0.546376	0.530925	0.518726	0.507454	0.495913
26	0.549817	0.526569	0.510973	0.498655	0.487254	0.475586
28	0.531617	0.508162	0.492471	0.48007	0.468576	0.456818
30	0.514609	0.491012	0.475265	0.462814	0.451255	0.439439
32	0.498679	0.474993	0.459224	0.446748	0.435149	0.4233
34	0.483729	0.459997	0.444234	0.431755	0.420136	0.408275
36	0.46967	0.44593	0.430196	0.417729	0.406107	0.394254
38	0.456425	0.432708	0.41702	0.404582	0.39297	0.381138
40	0.443925	0.420256	0.404631	0.392232	0.380642	0.368845
42	0.432109	0.40851	0.392959	0.38061	0.369052	0.3573
44	0.420922	0.397411	0.381945	0.369653	0.358136	0.346438
46	0.410315	0.386907	0.371534	0.359305	0.347836	0.336199
48	0.400244	0.376951	0.361678	0.349519	0.338102	0.326533
50	0.390669	0.367502	0.352335	0.340248	0.328889	0.317393
52	0.381555	0.358522	0.343464	0.331454	0.320157	0.308738
54	0.372868	0.349977	0.335031	0.323101	0.311868	0.30053
56	0.36458	0.341835	0.327005	0.315157	0.303991	0.292736
58	0.356664	0.33407	0.319357	0.307591	0.296495	0.285327
60	0.349094	0.326655	0.312061	0.300379	0.289354	0.278273
62	0.341849	0.319567	0.305092	0.293496	0.282543	0.271552



In the following graph the deflection coefficient on the lagging is shown positive at the lagging to see how the distribution is for various *x/b*. We see the deflection at the column is practically zero for *t/b* = 0.1.

$$\frac{EI\,v'(x)}{\frac{pb^3}{3}} = \frac{3}{\Gamma}\sum_{n=1}^{\infty}\frac{2\frac{1+\Gamma}{\pi n}+\frac{\pi n}{1+\Gamma}\frac{C2}{0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3}+A}\sin\alpha_n b\sin\alpha_n x \quad \dots\dots\dots\dots\dots 96$$





The deflection does not take into account the moment of inertia of the elastic media, so it is conservative. But for a media such as soils where tension is not allowed it is accurate and useful.

Once D id found the stresses at $q = 0$ and $h = \infty$ are the same as before only C2 is different:

$$\sigma_y = -\sum_{n=\varphi}^{\infty} D\cos\alpha_n x\left(1 + \frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y} - p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 97$$

$$D = -p\frac{A}{\Gamma}\left[\frac{2\dfrac{(1+\Gamma)}{\pi n} + \dfrac{\pi n}{(1+\Gamma)}\dfrac{C2}{0.5pb^2}}{\dfrac{(\pi n)^3}{(1+\Gamma)^3} + A}\right]\sin\alpha_n b \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..98$$

$$\sigma_x = k_0\sum_{n=\varphi}^{\infty} D\cos\alpha_n x\left(1 + \frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y} - 2k_0\sum_{n=\varphi}^{\infty} De^{-\alpha_n y}\cos\alpha_n x - k_0 p \ldots\ldots\ldots\ldots 99$$

$$\tau_{xy} = -\sum_{n=\varphi}^{\infty}\frac{\pi n}{1+\Gamma}D\frac{y}{b}e^{-\alpha_n y}\sin\alpha_n x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.100$$

Example 2:

We redo example 1 with fixed end condition:

	Example in article the strain at y=0 is zero						
			b=	2 ft			4 ft opening
			t =	1	ft	s =	3 ft
	Concrete	150 pcf	at the powerhouse at the dam				
p=	13500	lbs/ft	30 ft height x 3 ft thick				
p=	1125	lbs/in	Pick t/b = 0.5				
p/ inch =	1125	psi	v=			0.2	
			We use ACI 318 11 code use Ft =				
Ft=	87.63561	psi	1.6 3000				
sFt/p/ =	0.3895	>0.067442	F'c =			3000	psi

$$M_n = 5\lambda\sqrt{f'_c}\,S_m \qquad (22\text{-}2)$$

$$V_n = 4/3\lambda\sqrt{f'_c}\,b_w h \qquad (22\text{-}9)$$

$$A = \frac{4}{(3-\nu)(1+\nu)}\left[\frac{sb^3 E_s}{2EI}\right]$$

Es = E

Thus	IA =		14.28571		
		for sFt/p =			
A =	141	0.077803		0.067442	
thus I =	0.101317	ft^4			
thus I =	2100.912	in^4	=	sh^3/12	
h^3 =	700.304		h =	8.880325	in
Sx =	473.161	in^3	A =	319.6917	in^2
1.3pb^2/3					
Mu = =	23400	Lbs ft		Ultimate Moment	
Mu reduced =	6042.493	k ft		Rm =	0.28453
ft = M/Sx	168.8558	< Ft =		273.8613	psi
Rv =	0.070038		At x = 0.63b		
1.3fv =	11.53453	> Fv =		73.02967	psi

So, the tension sFt/p will act upward on the beam with 0.067442 at the center. The beam is ok in compression also.

If we don't allow tension in concrete

sFt/p =	0.30883				
Use compression at A = 20				Nominal	
Thus	IA =		14.28571		
A =	20				
thus I =	0.714286	ft^4			
thus I =	14811.43	in^4	=	sh^3/12	
h^3 =	4937.143		h =	17.0278	in
Sx =	1739.676	in^3	A =	613.0008	in^2
1.3pb^2/3					
Mu = =	23400	Lbs ft		Ultimate Moment	
Mu reduced =	15171.93	k ft		Rm =	0.648373
ft = M/Sx	104.6535	< Ft =		273.8613	psi OK
Rv =	0.290508	At x = 0.291b			
1.3fv =	24.95145	< Fv =		73.02967	psi OK

So, *Rm*- and *Rv* was found for *A* = 20 from the tables. So, we can see that the opening can be installed. The infinite height used as an assumption should be reduced by 17.0278 inch which does not change the solution.

Closed form solution #3 simple span beam with height *h*:
When starting with equation 24 this process can be repeated where *s/2β is* skn

The load on the lagging and reaction at $y = 0$ is

$$\sigma_y = -\sum_{n=1}^{\infty}\left(\alpha_n s\frac{k_n}{\beta}v_n + W_n\right)\cos\alpha_n x - p \quad \text{for } -b \le x \le b \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..100$$

The reaction is

$$\sigma_y = -\sum_{n=1}^{\infty}\left(\alpha_n s\frac{k_n}{\beta}v_n + W_n\right)\cos\alpha_n x + p\frac{b}{t} + q\frac{b}{t} \quad \text{for } b \le x \le b+t \ldots\ldots\ldots\ldots\ldots\ldots..\ 101$$

Thus

$$\sigma_y = \sum_{n=1}^{\infty}\left[-\left(\alpha_n s\frac{k_n}{\beta}v_n + W_n\right)\cos\alpha_n x - \frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n}\cos\alpha_n x - \frac{2qb}{t(b+t)}\frac{\sin\alpha_n b}{\alpha_n}\cos\alpha_n x\ \right] + \frac{2qb}{b+t}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.102$$

We note wn is Fourier cosine for the load *q* on the beam at *y* = *h* thus

$$\sum_{n=\emptyset}^{\infty} w_n\cos\alpha_n x = -\sum_{n=1}^{\infty}\frac{2q}{(b+t)}\frac{\sin\alpha_n b}{\alpha_n}\cos\alpha_n x - \frac{2qb}{b+t}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.103$$

$$w_n = \frac{-2q}{(b+t)}\frac{\sin\alpha_n b}{\alpha_n} = -2q\frac{\sin\alpha_n b}{\pi n}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots104$$

We find the constant term in equation 102 and 103 is for all *x* thus they cannot produce arching and they cancel each other. Thus, they are dropped, and the shear is

$$\int\sigma_y = -\sum_{n=1}^{\infty}\left(s\frac{k_n}{\beta}v_n + \frac{W_n}{\alpha_n}\right)\sin\alpha_n x - \frac{2p}{t}\sum_{n=1}^{\infty}\frac{\sin\alpha_n b}{\alpha_n^2}\sin\alpha_n x - q\frac{b}{t}\frac{2}{b+t}\sum_{n=1}^{\infty}\frac{\sin\alpha_n b}{\alpha_n^2}\sin\alpha_n x\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.105$$

or

$$\int\sigma_y = -\sum_{n=1}^{\infty}\left(s\frac{k_n}{\beta}v_n\right)\sin\alpha_n x - \sum_{n=1}^{\infty}\left(\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^2} + \frac{W_n}{\alpha_n} + 2q\frac{b}{t(b+t)}\frac{\sin\alpha_n b}{\alpha_n^2}\right)\sin\alpha_n x$$

Thus

$$v_n = -\frac{\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^5} + \frac{W_n}{\alpha_n^4} + 2q\frac{b}{t(b+t)}\frac{\sin\alpha_n b}{\alpha_n^5} + \frac{2C2}{t}\frac{\sin\alpha_n b}{\alpha_n^3}}{EI + \frac{sk_n}{\beta\alpha_n^3}} \quad \dots\dots\dots\dots\dots\dots\dots\dots.106$$

Finding v0 set the equation to

$$EIv_n = -\frac{\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^5} + \frac{W_n}{\alpha_n^4} + 2q\frac{b}{t(b+t)}\frac{\sin\alpha_n b}{\alpha_n^5} + \frac{2C2}{t}\frac{\sin\alpha_n b}{\alpha_n^3}}{1 + \frac{sk_n}{\beta EI\alpha_n^3}}$$

$$EIv_n = -\frac{\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^2} + \frac{W_n}{\alpha_n} + 2q\frac{b}{t(b+t)}\frac{\sin\alpha_n b}{\alpha_n^2} + \frac{2C2}{t}\sin\alpha_n b}{\alpha_n^3 + \frac{sk_n}{\beta EI}}$$

At $n = 0$ is the same as saying $h = 0$ and we have no medium. $kn = 0$ $Wn = -wn$. and v0 = C4 = ∞. Again, making the argument v0 and C4 will be calculated when setting the deflection to zero at ±*b*.

$$\iint \sigma_y = \sum_{n=1}^{\infty}\left(\frac{sk_n v_n}{\beta\alpha_n}\right)\cos\alpha_n x + \sum_{n=1}^{\infty}\left(\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^3} + \frac{W_n}{\alpha_n^2} + 2q\frac{b}{t(b+t)}\frac{\sin\alpha_n b}{\alpha_n^3}\right)\cos\alpha_n x + C2$$

$$\iint \sigma_y = -\sum_{n=1}^{\infty}\left(\frac{sk_n}{\beta EI\alpha_n}\right)\frac{\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^2} + \frac{W_n}{\alpha_n} + 2q\frac{b}{t(b+t)}\frac{\sin\alpha_n b}{\alpha_n^2} + \frac{2C2}{t}\sin\alpha_n b}{\alpha_n^3 + \frac{sk_n}{\beta EI}}\cos\alpha_n x + \sum_{n=1}^{\infty}\left(\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^3} + \frac{W_n}{\alpha_n^2} + 2q\frac{b}{t(b+t)}\frac{\sin\alpha_n b}{\alpha_n^3}\right)\cos\alpha_n x + C2$$

$$\iint \sigma_y = -\sum_{n=1}^{\infty}\left(\frac{sk_n}{\beta EI}\right)\frac{\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^3} + \frac{W_n}{\alpha_n^2} + 2q\frac{b}{t(b+t)}\frac{\sin\alpha_n b}{\alpha_n^3} + \frac{2C2}{t\alpha_n}\sin\alpha_n b}{\alpha_n^3 + \frac{sk_n}{\beta EI}}\cos\alpha_n x + \sum_{n=1}^{\infty}\left(\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^3} + \frac{W_n}{\alpha_n^2} + 2q\frac{b}{t(b+t)}\frac{\sin\alpha_n b}{\alpha_n^3}\right)\cos\alpha_n x + C2$$

$$\iint \sigma_y = \sum_{n=1}^{\infty} \frac{\left[\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^3} + \frac{W_n}{\alpha_n^2} + 2q\frac{b}{t(b+t)}\frac{\sin\alpha_n b}{\alpha_n^3}\right]\alpha_n^3 - \left(\frac{sk_n}{\beta EI\alpha_n}\right)\frac{2C2}{t}\sin\alpha_n b}{\alpha_n^3 + \frac{sk_n}{\beta EI}}\cos\alpha_n x + $$

$$+ C2$$

$$\iint \sigma_y = \sum_{n=1}^{\infty} \frac{\frac{2p}{t}\sin\alpha_n b + \alpha_n W_n + 2q\frac{b}{t(b+t)}\sin\alpha_n b - \left(\frac{sk_n}{\beta EI\alpha_n}\right)\frac{2C2}{t}\sin\alpha_n b}{\alpha_n^3 + \frac{sk_n}{\beta EI}}\cos\alpha_n x$$

$$+ C2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. 107$$

At $x = b$ the moment is zero

$$\sum_{n=1}^{\infty} \frac{\frac{2p}{t}\sin\alpha_n b + \alpha_n W_n + 2q\frac{b}{t(b+t)}\sin\alpha_n b}{\alpha_n^3 + \frac{sk_n}{\beta EI}}\cos\alpha_n b = -C2 + \frac{\left(\frac{sk_n}{\beta EI\alpha_n}\right)\frac{2C2}{t}}{\alpha_n^3 + \frac{sk_n}{\beta EI}}\sin\alpha_n b$$

$$C2 = \frac{\sum_{n=1}^{\infty} \frac{\frac{2p}{t}\sin\alpha_n b + \alpha_n W_n + 2q\frac{b}{t(b+t)}\sin\alpha_n b}{\alpha_n^3 + \frac{sk_n}{\beta EI}}\cos\alpha_n b}{-1 + \frac{\left(\frac{sk_n}{\beta EI\alpha_n}\right)\frac{2}{t}}{\alpha_n^3 + \frac{sk_n}{\beta EI}}\sin\alpha_n b}$$

$$C2 = \frac{b^3 \sum_{n=1}^{\infty} \frac{\frac{2p}{\Gamma b}\sin\alpha_n b + \frac{\pi n}{(1+\Gamma)b}W_n + 2q\frac{1}{\Gamma(1+\Gamma)b}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3} + \frac{sk_n b^3}{\beta EI}}\cos\alpha_n b}{-1 + \frac{\left(\frac{sk_n b^3}{\beta EI\alpha_n}\right)\frac{2}{t}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + \frac{sk_n b^3}{\beta EI}}\sin\alpha_n b}$$

$$\frac{C2}{0.5b^2} = \frac{-\sum_{n=1}^{\infty} \frac{\left\{4p\sin\alpha_n b + \frac{2\pi n\Gamma}{(1+\Gamma)} W_n + \frac{4q}{(1+\Gamma)}\sin\alpha_n b\right\}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}\cos\alpha_n b}{\sum_{n=1}^{\infty} 2k_n A \frac{2(1+\Gamma)\frac{\sin\alpha_n b}{\pi n}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}\cos\alpha_n b - \Gamma} \ldots\ldots\ldots\ldots\ldots\ldots\ldots 108$$

And the moment is

$$\iint \sigma_y = \sum_{n=1}^{\infty} \frac{\frac{2p}{t}\sin\alpha_n b + \alpha_n W_n + 2q\frac{b}{t(b+t)}\sin\alpha_n b - \left(\frac{sk_n}{\beta EI\alpha_n}\right)\frac{2C2}{t}\sin\alpha_n b}{\alpha_n^3 + \frac{sk_n}{\beta EI}}\cos\alpha_n x + C2$$

$$\frac{\iint \sigma_y}{0.5b^2} = \sum_{n=1}^{\infty} \frac{1}{\Gamma}\left(\frac{4p\sin\alpha_n b + \frac{2\pi n\Gamma}{(1+\Gamma)}W_n + \frac{4q}{(1+\Gamma)}\sin\alpha_n b - 4k_n A\frac{C2}{0.5b^2}\frac{(1+\Gamma)}{\pi n}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}\right)\cos\alpha_n x + \frac{C2}{0.5b^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 109$$

And the shear is,

$$\int \sigma_y = \sum_{n=1}^{\infty} 2k_n A\left(\frac{\frac{2pb(1+\Gamma)^2}{\Gamma}\frac{\sin\alpha_n b}{(\pi n)^2} + \frac{W_n b}{n\pi}(1+\Gamma) + \frac{2qb(1+\Gamma)}{\Gamma}\frac{\sin\alpha_n b}{(\pi n)^2} + \frac{b}{\Gamma}\frac{C2}{0.5b^2}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}\right)\sin\alpha_n x - \sum_{n=1}^{\infty}\left(\frac{2pb(1+\Gamma)^2}{\Gamma}\frac{\sin\alpha_n b}{(\pi n)^2} + \frac{W_n b}{n\pi}(1+\Gamma) + \frac{2qb(1+\Gamma)}{\Gamma}\frac{\sin\alpha_n b}{(\pi n)^2}\right)\sin\alpha_n x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. 110$$

And the stress is,

$$\sigma_y = \sum_{n=1}^{\infty} 2k_n A \left(\frac{\frac{2p(1+\Gamma)}{\Gamma}\frac{\sin\alpha_n b}{n\pi} + W_n + \frac{2q}{\Gamma}\frac{\sin\alpha_n b}{n\pi} + \frac{\pi n}{\Gamma(1+\Gamma)}\frac{C2}{0.5b^2}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A} \right)\cos\alpha_n x - \sum_{n=1}^{\infty}\left(\frac{2p(1+\Gamma)}{\Gamma}\frac{\sin\alpha_n b}{n\pi} + W_n + \frac{2q}{\Gamma}\frac{\sin\alpha_n b}{n\pi}\right)\cos\alpha_n x \ldots\ldots\ldots\ldots\ldots\ldots\ldots.111$$

Thus

Again, when *I* is infinite or *Es* is zero as in water, we have C2 with Wn = wn,

$$C2 = -\sum_{n=1}^{\infty}\left(\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^3} + 2q\frac{b}{t(b+t)}\frac{\sin\alpha_n b}{\alpha_n} + \frac{2q}{(b+t)}\frac{\sin\alpha_n b}{\alpha_n}\right)\cos\alpha_n b$$
$$= -\sum_{n=1}^{\infty}\left(\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^3} + \frac{2q}{t}\frac{\sin\alpha_n b}{\alpha_n^3}\right)\cos\alpha_n b = -(p+q)\frac{b^2}{2} \;\; in - b \le x \le b$$

We will set first *q* to zero and calculate *Rm Rv* and the stress ratio. Now *A* can be zero for infinite moment of inertia, but Es cannot be zero because of Hooke's law does not work for water in the deflection. We find there is live load stress distribution for *A* = 0 because of the simple span requirements and *I* do not enter the stress in the right side of equation 22 and in this case the stress is when *A* is zero for small *s*. When we do the fixed end solution, which is more realistic, the stress at *A* = 0 is a different distribution. There is a region for *t*/*b* where the thickness of the column causes more arching meaning a thinner column cause more arching for some *t*/*b.* This means a thinner column for some *t*/*b* sucks up more load because it becomes a hard spot with deflection close to zero. This is where the deflection at the column makes a difference. When *t*/*b* is large a thicker column causes more arching or sucks up more load.

Thus, at $q = 0$

$$\frac{C2\;1}{0.5pb^2} = \frac{\sum_{n=1}^{\infty}\frac{4\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3}+2k_nA}\cos\alpha_n b}{\sum_{n=1}^{\infty} 2k_nA\frac{2(1+\Gamma)\frac{\sin\alpha_n b}{\pi n}}{\frac{(\pi n)^3}{(1+\Gamma)^3}+2k_nA}\cos\alpha_n b - \Gamma} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.112$$

and $x = 0$ we have
Note: the maximum moment may not be at $x = 0$ depending on *A*

$$Rm1 = \frac{\iint \sigma_y}{0.5pb^2} = \frac{1}{\Gamma}\sum_{n=1}^{\infty} \frac{4\sin\alpha_n b - 4k_n A\left(\frac{C2}{0.5pb^2}\right)\frac{(1+\Gamma)}{\pi n}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A} + \frac{C2}{0.5pb^2} \dots\dots\dots\dots 113$$

Table 10 Rm 1 verses h/b for = 0.1

h/b	A = 0	A = 5	A = 10	A = 15	A = 20	A = 25	A = 30
1E 13	1	1	1	1	1	1	1
0.1	1	0.996449	0.992929	0.989439	0.985978	0.982547	0.979145
0.2	1	0.975475	0.952305	0.930378	0.909593	0.88986	0.871099
0.3	1	0.933979	0.876881	0.826976	0.782958	0.74382	0.708774
0.4	1	0.876778	0.78172	0.70609	0.644434	0.593167	0.549838
0.5	1	0.813006	0.685413	0.592778	0.522453	0.467235	0.42272
0.6	1	0.752675	0.602241	0.501212	0.428762	0.37432	0.331949
0.7	1	0.702067	0.537632	0.433687	0.362221	0.310186	0.270685
0.8	1	0.66298	0.49062	0.386386	0.316823	0.267269	0.230292
0.9	1	0.634414	0.457733	0.354168	0.286449	0.238923	0.203869
1	1	0.614298	0.435278	0.332568	0.26633	0.220305	0.186624
1.5	1	0.578338	0.396517	0.296022	0.232726	0.189489	0.158269
2	1	0.574166	0.392131	0.291944	0.229011	0.186103	0.155168

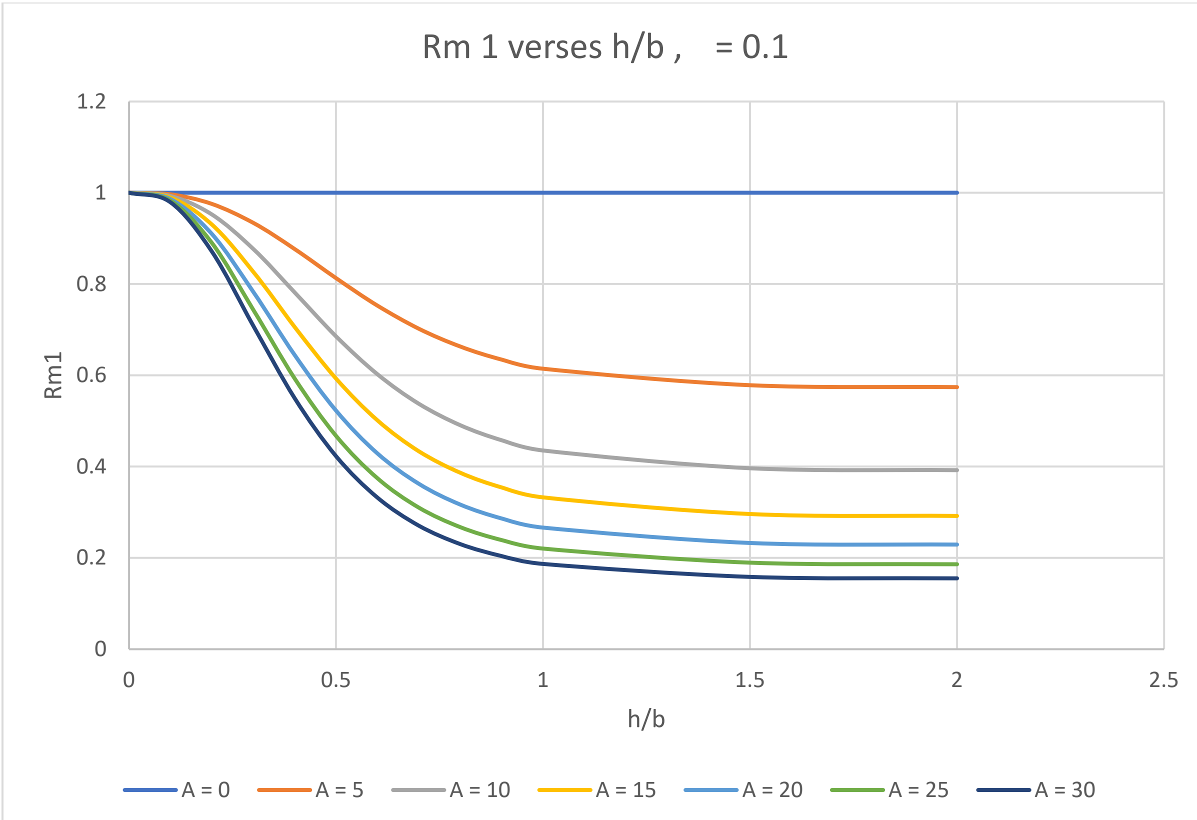
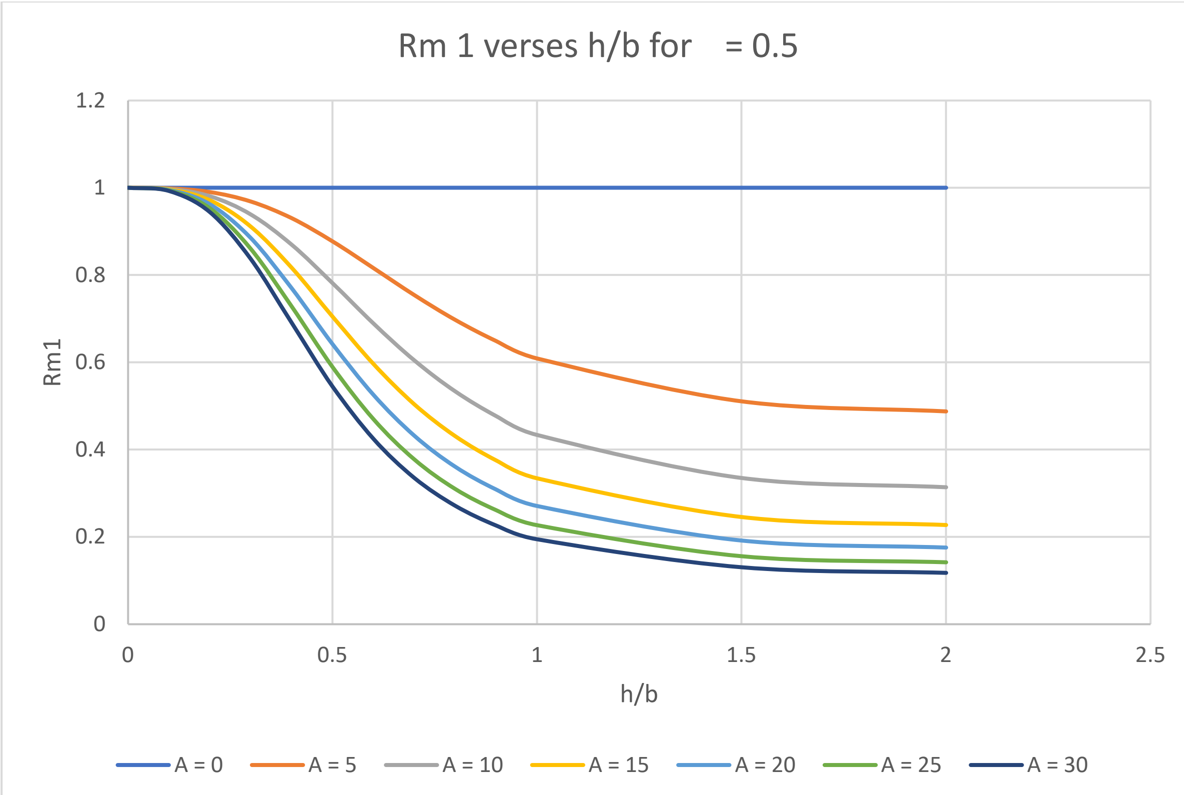


Table 11 Rm 1 verses h/b for   = 0.5

h/b	A = 0	A = 5	A = 10	A = 15	A = 20	A = 25	A = 30
1E 13	1	1	1	1	1	1	1
0.1	1	0.998757	0.997516	0.996279	0.995045	0.993814	0.992585
0.2	1	0.990256	0.980702	0.971331	0.962138	0.953119	0.944268
0.3	1	0.96847	0.93889	0.911082	0.884891	0.860179	0.836823
0.4	1	0.930241	0.869694	0.816639	0.76976	0.728033	0.690648
0.5	1	0.877276	0.781566	0.704822	0.641904	0.589377	0.544858
0.6	1	0.815961	0.689069	0.596299	0.525528	0.469766	0.4247
0.7	1	0.753995	0.604342	0.503795	0.431645	0.377386	0.335122
0.8	1	0.697228	0.533319	0.430758	0.360647	0.309761	0.271193
0.9	1	0.648664	0.476936	0.375435	0.308569	0.261302	0.226186
1	1	0.609013	0.433588	0.334387	0.270825	0.226756	0.194493
1.5	1	0.510505	0.335085	0.245555	0.191601	0.155743	0.130316
2	1	0.48759	0.313862	0.227177	0.175613	0.141649	0.117729



Thus, at $q = 0$ and $x = b^*$ we have

$$Rv1 = \frac{1}{\Gamma}\sum_{n=1}^{\infty}\frac{2\frac{\pi n}{(1+\Gamma)} - 2k_n A\frac{C2}{0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}\sin\alpha_n b \sin\alpha_n b^* \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.114$$

Table 12 Rv 1 verses h/b for = 0.1 at b = b*

h/b	A = 0	A = 5	A = 10	A = 15	A = 20	A = 25	A = 30
1E 13	0.999755	0.999755	0.999755	0.999755	0.999755	0.999755	0.999755
0.1	0.999755	0.980388	0.96121	0.942218	0.92341	0.904783	0.886335
0.2	0.999755	0.941074	0.88597	0.83414	0.785314	0.73925	0.695732
0.3	0.999755	0.908958	0.831256	0.764081	0.705492	0.653995	0.608422
0.4	0.999755	0.886445	0.799749	0.731367	0.676118	0.630602	0.592494
0.5	0.999755	0.871046	0.783078	0.719088	0.670398	0.632068	0.601079
0.6	0.999755	0.860401	0.774357	0.715577	0.67263	0.639708	0.613543
0.7	0.999755	0.852953	0.76961	0.715185	0.676407	0.647082	0.623927
0.8	0.999755	0.847742	0.766876	0.715675	0.679734	0.652726	0.631427
0.9	0.999755	0.844131	0.765223	0.71628	0.682225	0.656703	0.636564
1	0.999755	0.84166	0.764187	0.716785	0.683972	0.659406	0.64
1.5	0.999755	0.837346	0.762532	0.717777	0.687024	0.664008	0.645767
2	0.999755	0.836851	0.762352	0.717896	0.68737	0.664521	0.646405

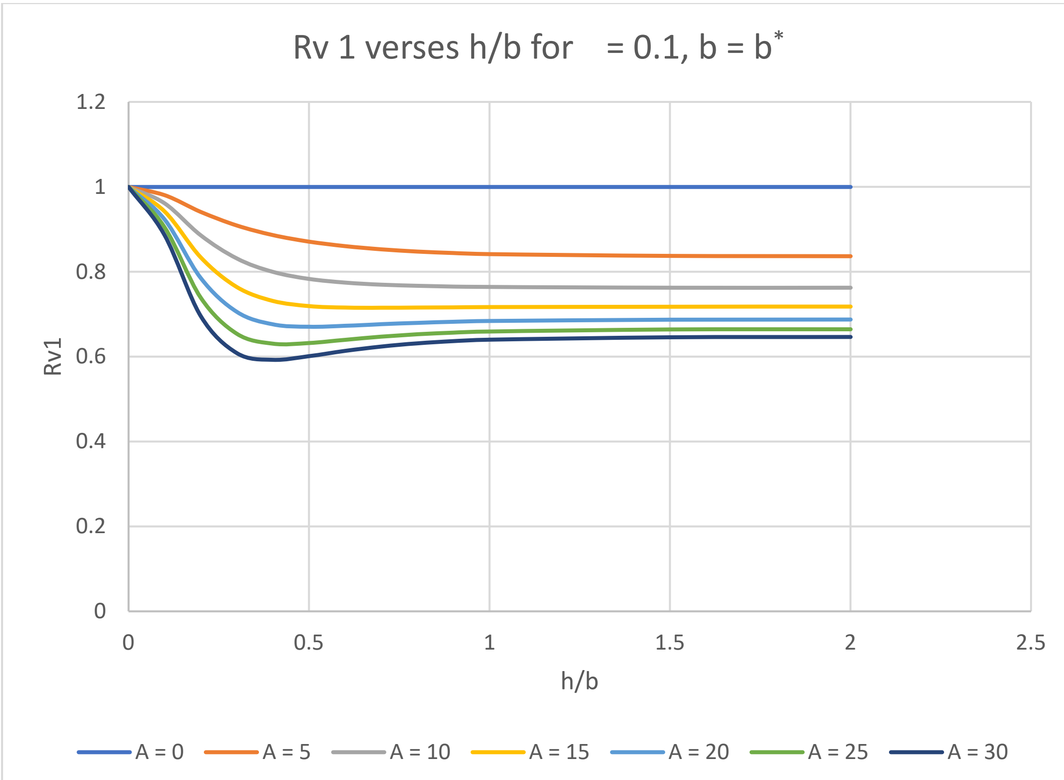
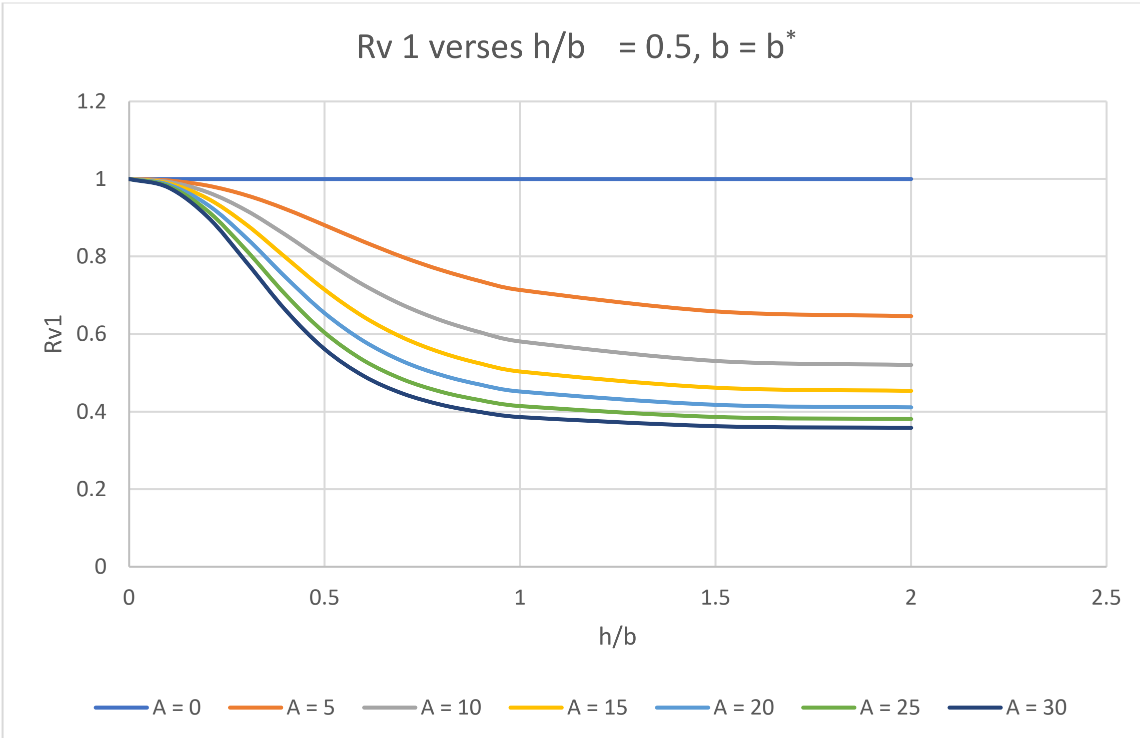


Table 13 Rv 1 verses h/b for = 0.5 b = b *

h/b	A = 0	A = 5	A = 10	A = 15	A = 20	A = 25	A = 30
1E 13	0.999909	0.999909	0.999909	0.999909	0.999909	0.999909	0.999909
0.1	0.999909	0.996247	0.992597	0.988959	0.985331	0.981716	0.978111
0.2	0.999909	0.982858	0.966192	0.949899	0.933968	0.918387	0.903146
0.3	0.999909	0.957919	0.918768	0.882191	0.847954	0.815848	0.785689
0.4	0.999909	0.922535	0.855827	0.797763	0.746801	0.70174	0.661634
0.5	0.999909	0.880926	0.788425	0.714489	0.654066	0.60378	0.561294
0.6	0.999909	0.838325	0.726542	0.644524	0.581719	0.532042	0.491733
0.7	0.999909	0.798776	0.675075	0.590965	0.529844	0.483278	0.446521
0.8	0.999909	0.764494	0.634697	0.551796	0.493866	0.450851	0.417483
0.9	0.999909	0.736173	0.60403	0.523665	0.469065	0.429206	0.398606
1	0.999909	0.713553	0.581133	0.503545	0.451866	0.414553	0.386082
1.5	0.999909	0.658659	0.530801	0.461854	0.417677	0.386358	0.362637
2	0.999909	0.646061	0.520186	0.453484	0.411046	0.381037	0.358316



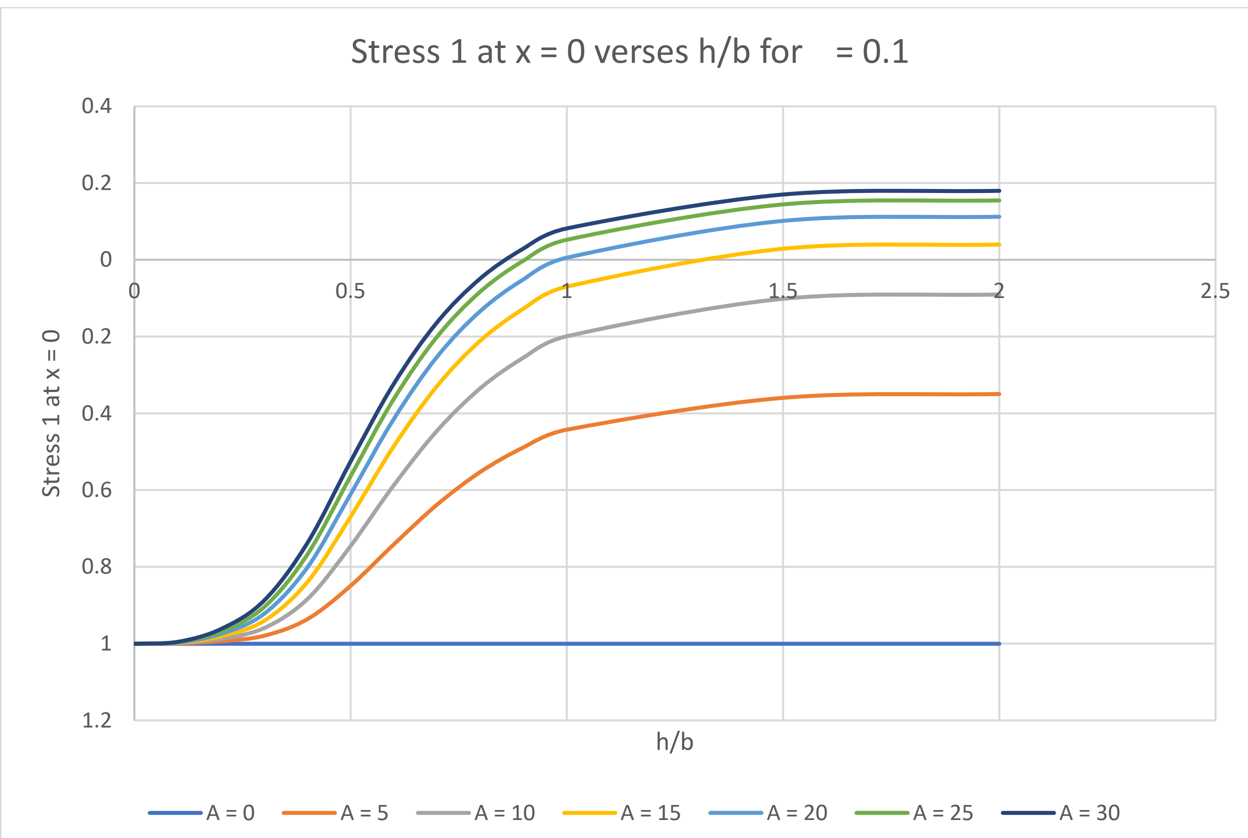
$$\frac{\sigma_y}{p} at\ x = 0 = \frac{sft1}{p} = \frac{1}{\Gamma}\sum_{n=1}^{\infty} 2k_n A \frac{2\frac{(1+\Gamma)}{\pi n} + \frac{\pi n}{(1+\Gamma)}\frac{C2}{0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A} \sin\alpha_n b - 1 \dots\dots\dots\dots\dots 115$$

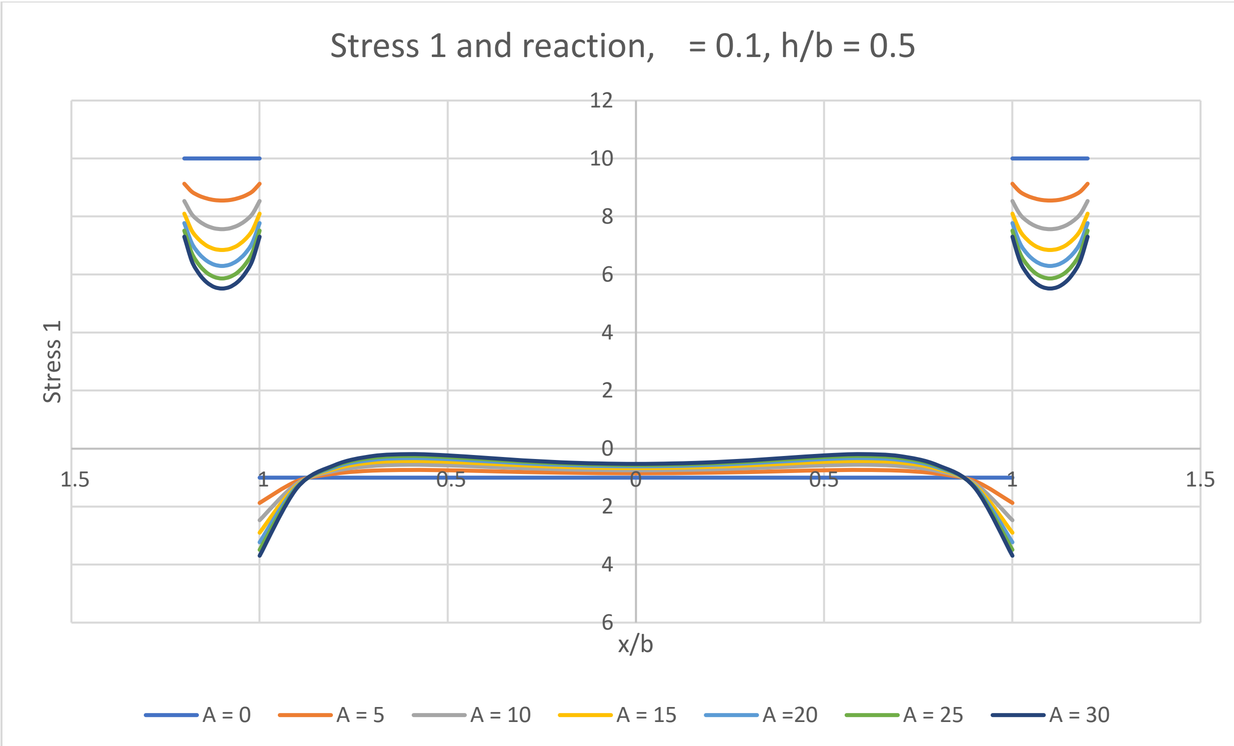
At the reaction:

$$at\ x = b + t = \frac{sft1}{p} = \frac{1}{\Gamma}\sum_{n=1}^{\infty} 2k_n A \frac{2\frac{(1+\Gamma)}{\pi n} + \frac{\pi n}{(1+\Gamma)}\frac{C2}{0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A} \sin\alpha_n b \cos\pi n + \frac{1}{\Gamma} \dots\dots\dots\dots 116$$

Table 14 Stress 1 at x = 0 verses h/b for   = 0.2   = 0.1

h/b	A = 0	A = 5	A = 10	A = 15	A = 20	A = 25	A = 30
1E 13	1	1	1	1	1	1	1
0.1	1	0.99917	0.99834	0.99751	0.99668	0.99585	0.99503
0.2	1	0.99344	0.98697	0.98057	0.97425	0.968	0.96183
0.3	1	0.97921	0.95937	0.94038	0.92217	0.90467	0.88783
0.4	1	0.93614	0.88374	0.83945	0.80117	0.76749	0.73745
0.5	1	0.84945	0.74559	0.66926	0.61054	0.56377	0.52549
0.6	1	0.74107	0.58666	0.48519	0.41409	0.36193	0.32231
0.7	1	0.63765	0.44506	0.32863	0.25254	0.20015	0.16275
0.8	1	0.55263	0.33472	0.21065	0.13354	0.08296	0.04859
0.9	1	0.4885	0.25472	0.12709	0.05055	0.00208	0.029568
1	1	0.44258	0.19903	0.06985	0.005723	0.052364	0.081927
1.5	1	0.35941	0.10136	0.028745	0.101572	0.144419	0.169988
2	1	0.34969	0.09022	0.039845	0.112276	0.154645	0.179733




Stress 1 and reaction, = 0.1, h/b = 0.5
Stress 1
x/b
A = 0
A = 5
A = 10
A = 15
A =20
A = 25
A = 30

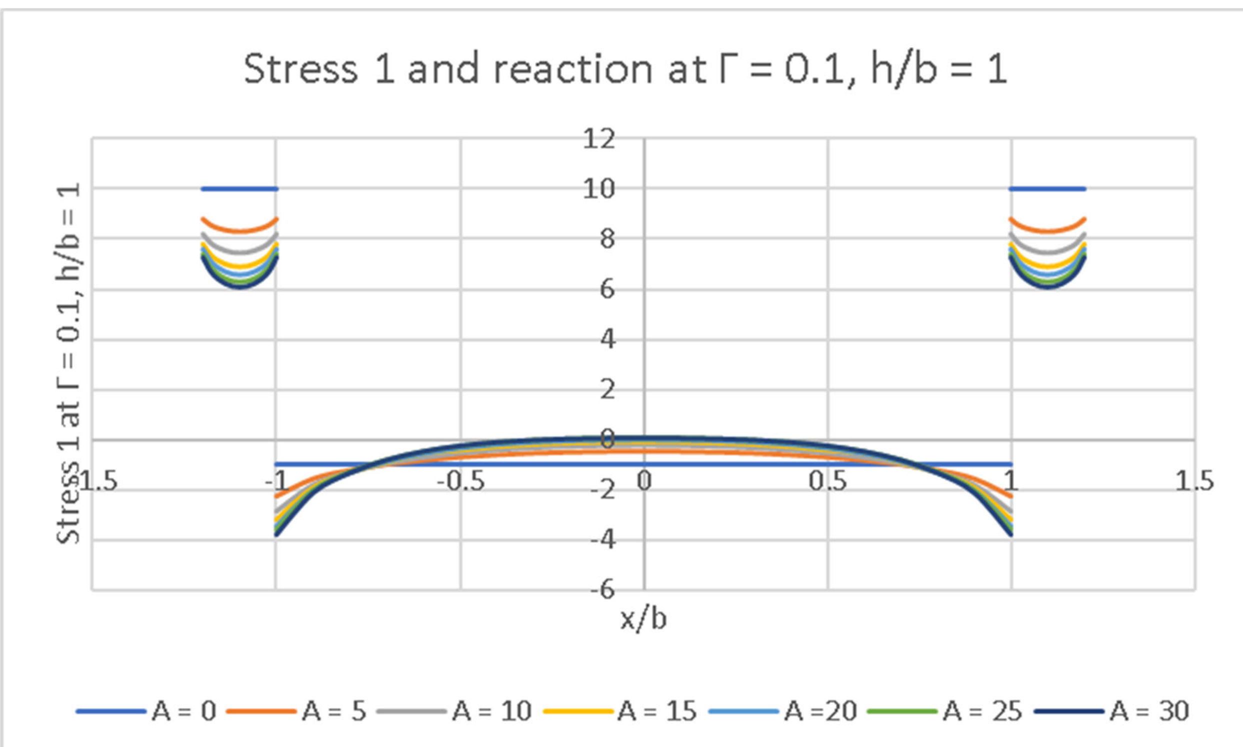
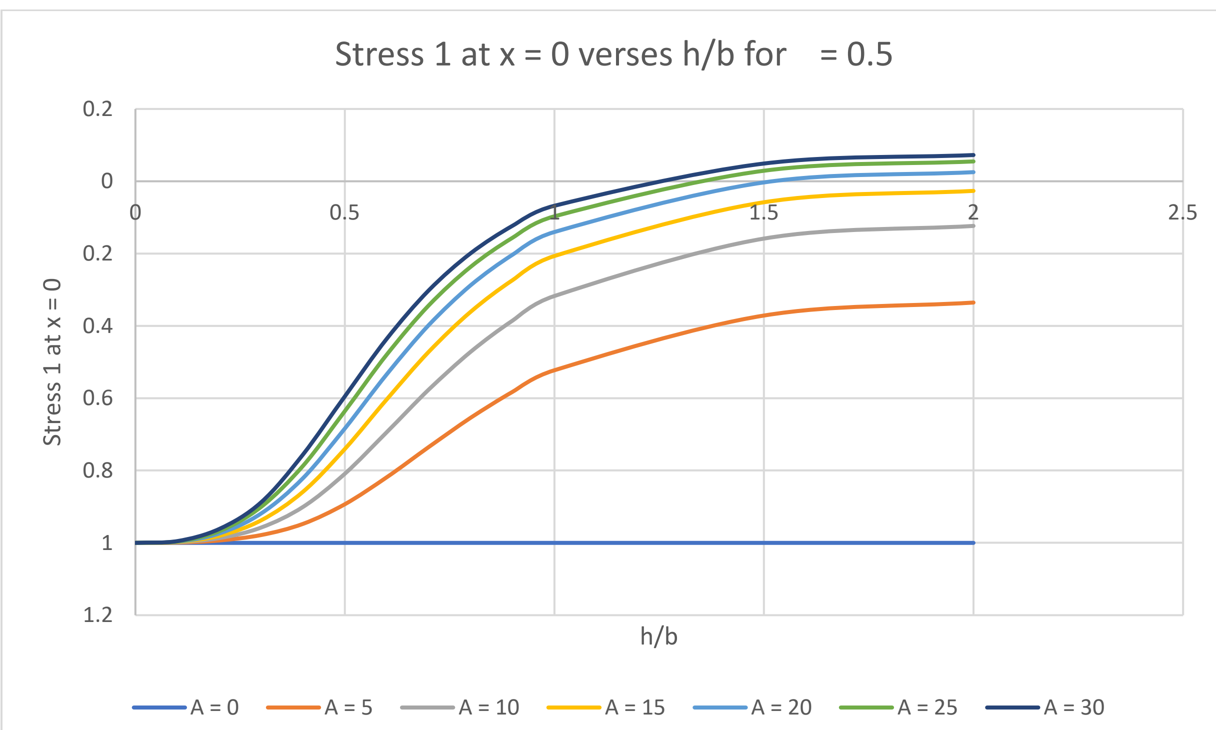
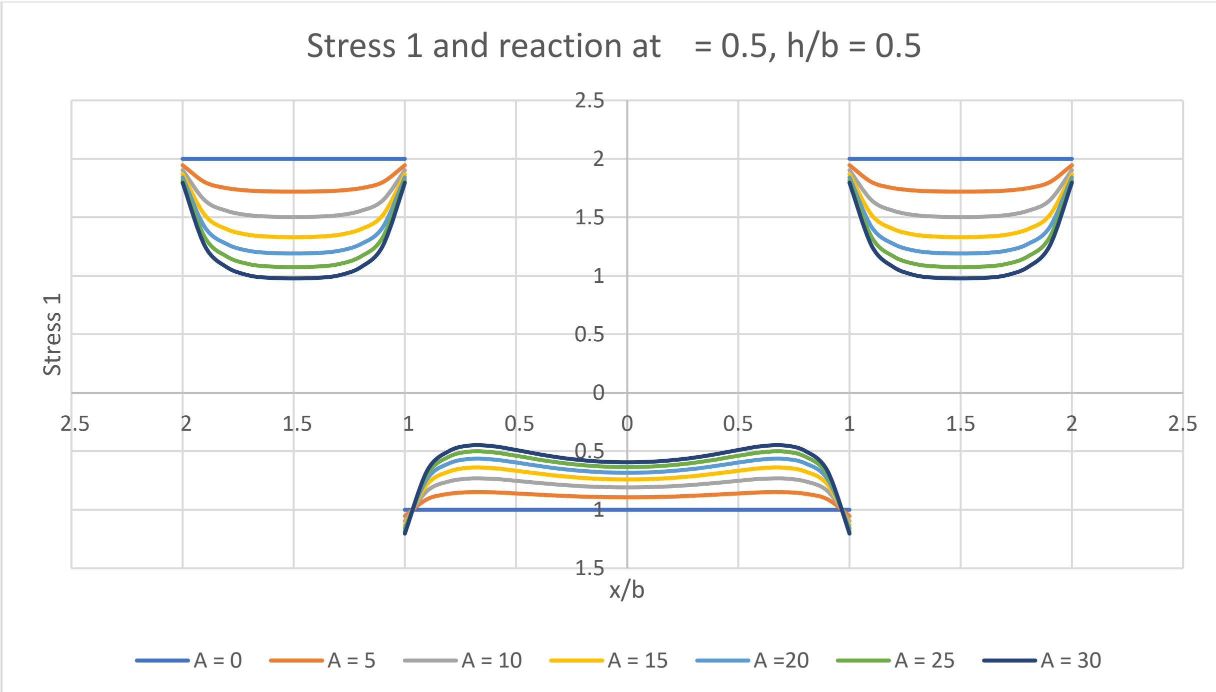
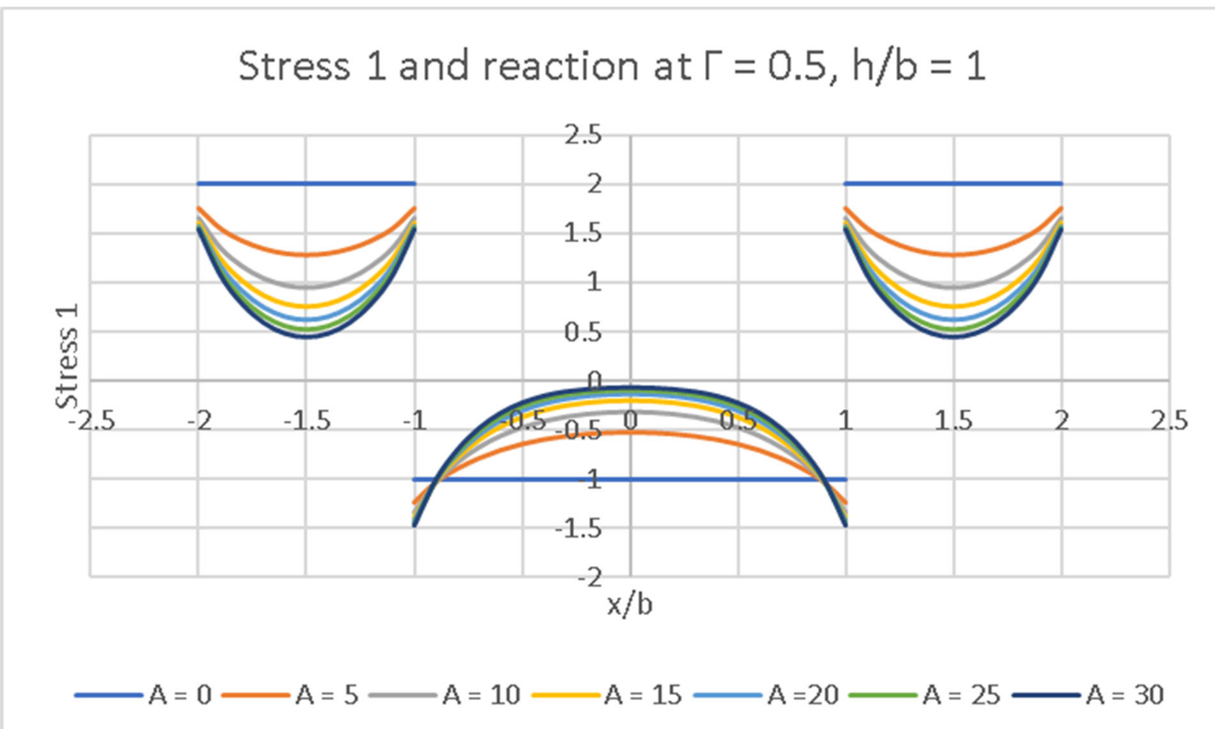

Stress 1 and reaction at Γ = 0.1, h/b = 1
Stress 1 at Γ = 0.1, h/b = 1
x/b
A = 0
A = 5
A = 10
A = 15
A =20
A = 25
A = 30

Table 15 Stress 1 at x = 0 verses h/b for = 0.5

h/b	A = 0	A = 5	A = 10	A = 15	A = 20	A = 25	A = 30
1E 13	1	1	1	1	1	1	1
0.1	1	0.99917	0.99834	0.99751	0.99668	0.99585	0.99502
0.2	1	0.99339	0.98686	0.98042	0.97406	0.96778	0.96158
0.3	1	0.97832	0.95759	0.93775	0.91873	0.90048	0.88783
0.4	1	0.94726	0.90048	0.85861	0.82085	0.78658	0.75529
0.5	1	0.89286	0.80838	0.73988	0.68311	0.63518	0.59412
0.6	1	0.81831	0.69346	0.6025	0.53332	0.47898	0.43517
0.7	1	0.73496	0.57634	0.47163	0.39787	0.34344	0.30184
0.8	1	0.65401	0.47164	0.36096	0.28779	0.23655	0.19913
0.9	1	0.58233	0.3852	0.27345	0.20325	0.15616	0.12309
1	1	0.5226	0.3171	0.20672	0.14012	0.09698	0.0677
1.5	1	0.37107	0.15826	0.05782	0.00298	0.029318	0.049101
2	1	0.3354	0.12349	0.0264	0.025341	0.055049	0.07268

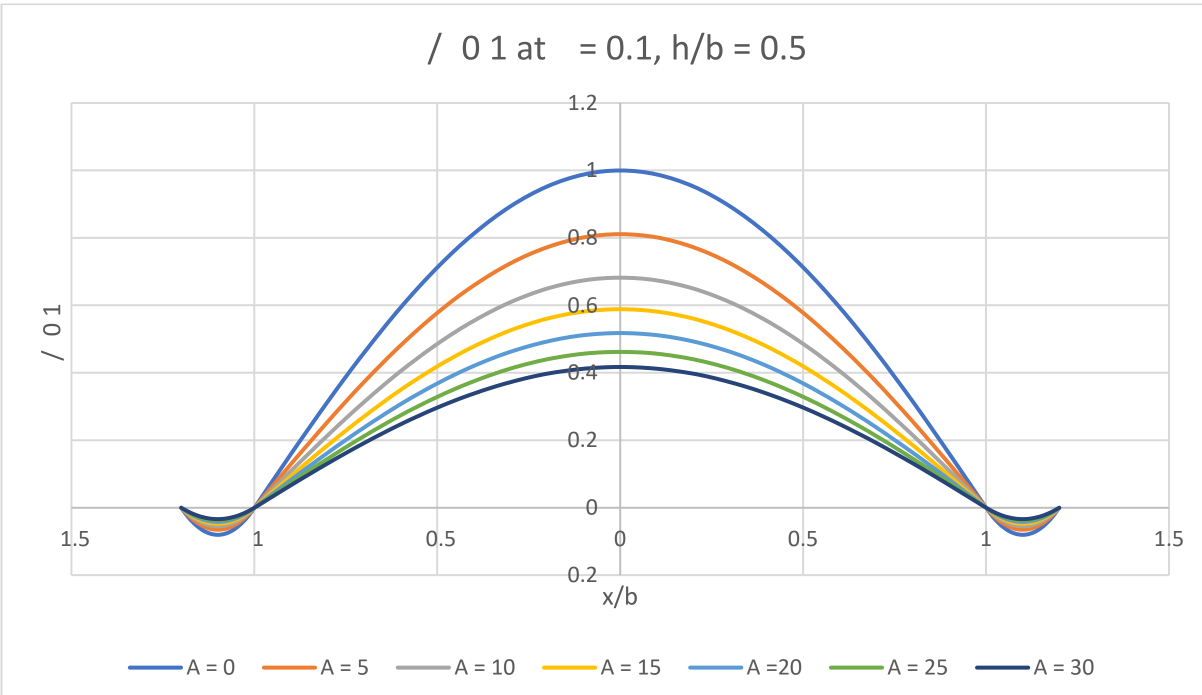



Stress 1 and reaction at = 0.5, h/b = 0.5
Stress 1
x/b
A = 0
A = 5
A = 10
A = 15
A =20
A = 25
A = 30

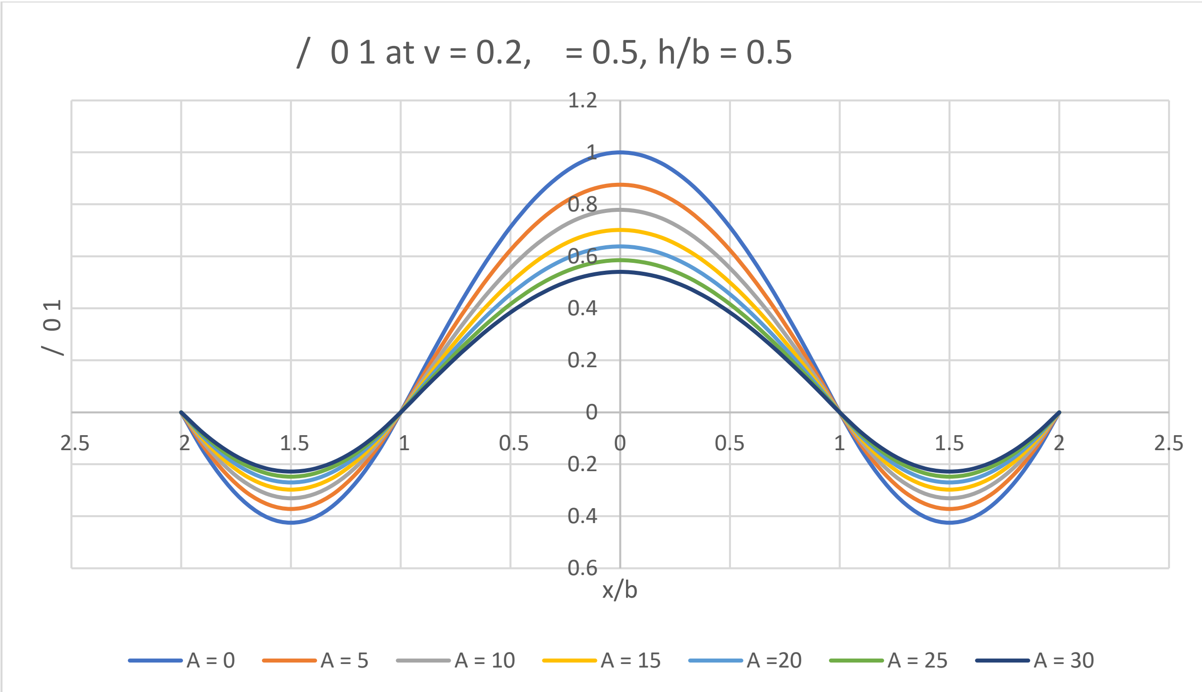

Stress 1 and reaction at Γ = 0.5, h/b = 1
Stress 1
x/b
A = 0
A = 5
A = 10
A = 15
A =20
A = 25
A = 30

$$EIv_n = -\frac{\frac{2p}{t}\frac{\sin\alpha_n b}{\alpha_n^2} + \frac{2C2}{t}\sin\alpha_n b}{\alpha_n^3 + \frac{sk_n}{\beta EI}}$$

$$\frac{EI\,v(x)1}{-\frac{5pb^4}{24}} = \frac{1}{\frac{5\Gamma}{24}} \sum_{n=1}^{\infty} \frac{2\frac{(1+\Gamma)^2}{(\pi n)^2} + \frac{C2}{0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_nA}(\cos\alpha_n x - \cos\alpha_n b)\sin\alpha_n b \ldots\ldots\ldots\ldots\ldots\ldots..117$$



The deflection does not take into account the moment of inertia of the elastic media, so it is conservative. But for a media such as soils where tension is not allowed it is accurate and useful.



$$\frac{EI\,v'(x)1}{\frac{pb^3}{3}} = \frac{3}{\Gamma}\sum_{n=1}^{\infty}\frac{2(1+\Gamma)\frac{\sin\alpha_n b}{\pi n}+\frac{C2}{0.5pb^2}\frac{\pi n}{(1+\Gamma)}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3}+2k_nA}\sin\alpha_n x \ldots\ldots\ldots\ldots..118$$

Once D is found the stresses with $q = 0$ are:

$$\sigma_y = -\sum_{n=\varphi}^{\infty} D\cos\alpha_n x\left[\left(a_n + b_n\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{\alpha_n y} + \left(c_n + \frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y}\right] - p\left(1-\frac{y}{h}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.\ldots119$$

$$\sigma_x = k_0\sum_{n=\varphi}^{\infty} D\cos\alpha_n x\left[\left(a_n + b_n\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{\alpha_n y} + \left(c_n + \frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y}\right] + 2k_0\sum_{n=\varphi}^{\infty} D\cos\alpha_n x\left[b_n e^{\alpha_n y} - e^{-\alpha_n y}\right] - k_0 p\left(1-\frac{y}{h}\right)\ldots\ldots\ldots\ldots\ldots\ldots120$$

$$\tau_{xy} = \sum_{n=\varphi}^{\infty} D\, sin\, \alpha_n\, x \left[\left(a_n + b_n\right) + b_n \frac{\pi n}{(1+\Gamma)} \frac{y}{b}\right) e^{\alpha_n y} + \left(-(c_n - 1) - \frac{\pi n}{(1+\Gamma)} \frac{y}{b}\right) e^{-\alpha_n y}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. 121$$

$$D = -\frac{s\alpha_n v_n}{\beta d_n} = p\left(\frac{sb^3}{\Gamma\beta EId_n}\right) \frac{\frac{2(1+\Gamma)}{\pi n} + \frac{\pi n}{(1+\Gamma)} \frac{C2}{0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A} \sin \alpha_n b$$

$$D = -\frac{s\alpha_n v_n}{\beta d_n} = p\left(\frac{2k_n A}{\Gamma d_n k_n}\right) \frac{\frac{2(1+\Gamma)}{\pi n} + \frac{\pi n}{(1+\Gamma)} \frac{C2}{0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A} \sin \alpha_n b$$

$$D = -p \frac{2k_n A}{\Gamma(a_n + c_n)} \frac{\frac{2(1+\Gamma)}{\pi n} + \frac{\pi n}{(1+\Gamma)} \frac{C2}{0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A} \sin \alpha_n b \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 122$$

$$a_n = -\left[c_n(1 + 2\alpha_n h) + 2\alpha_n^2 h^2\right] e^{-2\alpha_n h}$$

$$b_n = [2c_n - 1 + 2\alpha_n h] e^{-2\alpha_n h}$$

$$c_n = 1 + \frac{2\alpha_n^2 h^2 e^{-2\alpha_n h}}{(1 - 2\alpha_n h) e^{-2\alpha_n h} - 1}$$

$$d_n = a_n - b_n - c_n - 1$$

$$k_n = -\frac{a_n + c_n}{d_n}$$

Rewrite for the excel sheet

$$\sigma_y = -\sum_{n=\varphi}^{\infty} Dcos\, \alpha_n\, x \left[\left(a_{n1} + b_{n1} \frac{\pi n}{(1+\Gamma)} \frac{y}{b}\right) e^{-\alpha_n (h-y)} + \left(c_n + \frac{\pi n}{(1+\Gamma)} \frac{y}{b}\right) e^{-\alpha_n y}\right] - p\left(1 - \frac{y}{h}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 123$$

$$\sigma_x = k_0 \sum_{n=\varphi}^{\infty} D\cos\alpha_n x \left[\left(a_{n1} + b_{n1}\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n(h-y)} + \left(c_n + \frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y}\right] + 2k_0 \sum_{n=\varphi}^{\infty} D\cos\alpha_n x\left[b_{n1}e^{-\alpha_n(h-y)} - e^{-\alpha_n y}\right] - k_0 p\left(1 - \frac{y}{h}\right) \ldots\ldots\ldots\ldots\ldots 124$$

$$\tau_{xy} = \sum_{n=\varphi}^{\infty} D\sin\alpha_n x\left[\left((a_{n1} + b_{n1}) + b_{n1}\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n(h-y)} + \left(-(c_n - 1) - \frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 125$$

$$a_{n1} = a_n e^{\alpha_n y} = -\left[c_n(1 + 2\alpha_n h) + 2\alpha_n^2 h^2\right]e^{-\alpha_n h}$$

$$b_{n1} = b_n e^{\alpha_n y} = [2c_n - 1 + 2\alpha_n h]e^{-\alpha_n h}$$

Setting p to zero we have:

$$\frac{C2\,2}{0.5qb^2} = \frac{-\sum_{n=1}^{\infty}\dfrac{\left\{\dfrac{2\pi n\Gamma}{(1+\Gamma)}\dfrac{W_n}{q} + \dfrac{4}{(1+\Gamma)}\sin\alpha_n b\right\}}{\dfrac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}\cos\alpha_n b}{\sum_{n=1}^{\infty} 2k_n A\dfrac{2(1+\Gamma)\dfrac{\sin\alpha_n b}{\pi n}}{\dfrac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}\cos\alpha_n b - \Gamma} \ldots\ldots\ldots\ldots\ldots\ldots\ldots 126$$

and $x = 0$ we have, note: the maximum moment may not be at $x = 0$ depending on A

$$Rm2 = \frac{\iint \sigma_y}{0.5qb^2} = \sum_{n=1}^{\infty}\frac{1}{\Gamma}\left(\frac{\dfrac{2\pi n\Gamma}{(1+\Gamma)}\dfrac{W_n}{q} + \dfrac{4}{(1+\Gamma)}\sin\alpha_n b - 4k_n A\dfrac{C2}{0.5qb^2}\dfrac{(1+\Gamma)}{\pi n}\sin\alpha_n b}{\dfrac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}\right) + \frac{C2}{0.5qb^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 127$$

Table 16 Rm2 verses h/b for   = 0.1

h/b	A = 0	A = 5	A = 10	A = 15	A = 20	A = 25	A = 30
1E 13	1	1	1	1	1	1	1
0.1	0.999966	0.996415	0.9928954	0.9894054	0.985945	0.982514	0.9791119
0.2	0.998738	0.974258	0.9511304	0.9292434	0.908496	0.888799	0.8700707
0.3	0.995767	0.930134	0.8733674	0.8237493	0.779981	0.741062	0.706209
0.4	0.991541	0.869724	0.7757292	0.7009294	0.639936	0.589208	0.5463235
0.5	0.985873	0.80228	0.676959	0.5859352	0.516802	0.462495	0.4186948
0.6	0.978705	0.737807	0.5912057	0.4926927	0.422002	0.368846	0.3274474
0.7	0.970476	0.682775	0.5238924	0.4233886	0.354236	0.303845	0.2655587
0.8	0.961877	0.639248	0.474148	0.3742339	0.307499	0.25992	0.2243827
0.9	0.953547	0.60645	0.4386096	0.34016	0.275736	0.230483	0.1970753
1	0.945923	0.582458	0.4136744	0.3167775	0.254243	0.210758	0.1789087
1.5	0.921602	0.533563	0.3662073	0.2736838	0.215392	0.175559	0.1467868
2	0.912914	0.524343	0.3582264	0.2667938	0.209354	0.170187	0.1419458

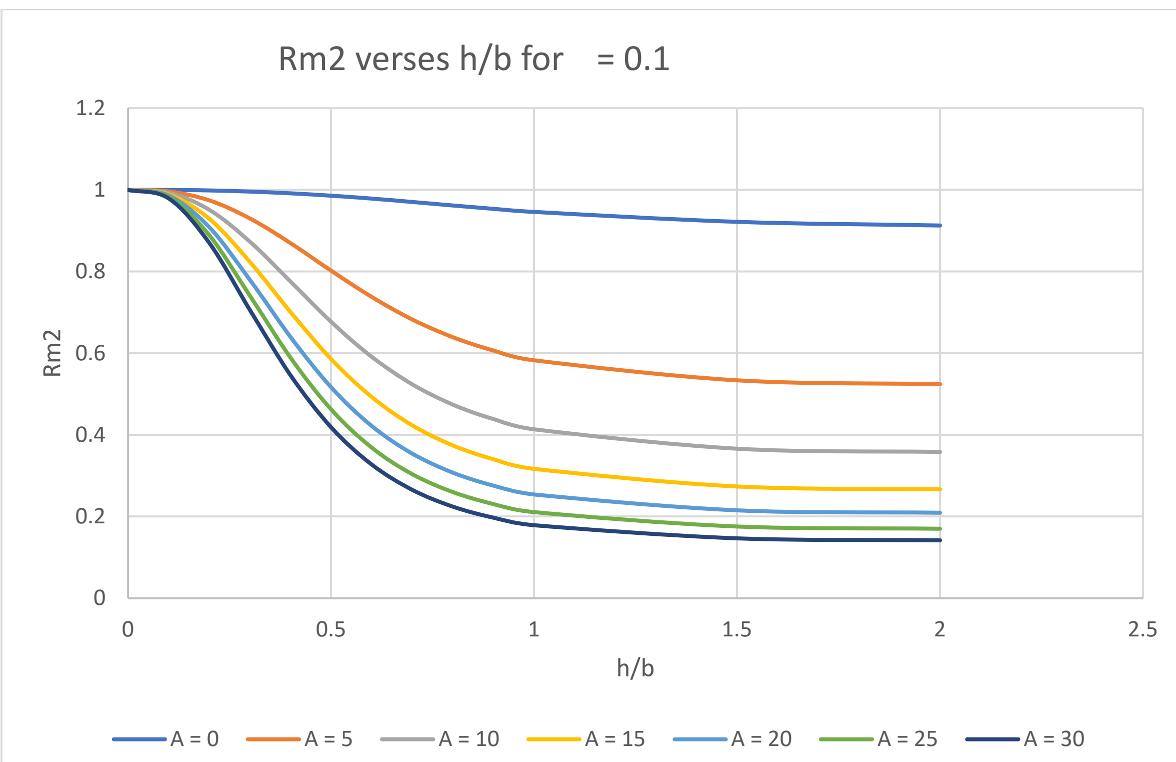
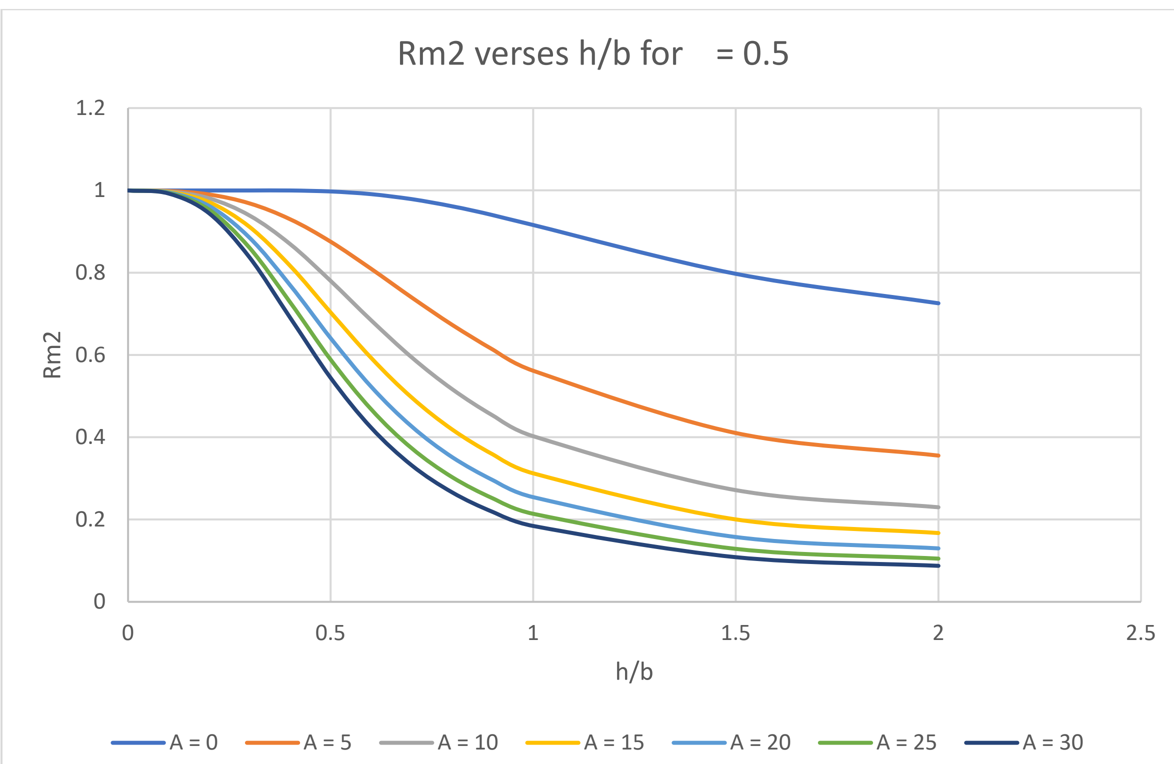


Table 17 Rm2 verses h/b for = 0.5

h/b	A = 0	A = 5	A = 10	A = 15	A = 20	A = 25	A = 30
1E 13	1	1	1	1	1	1	1
0.1	1	0.998757	0.9975162	0.9962789	0.995045	0.993814	0.9925855
0.2	1.000001	0.990257	0.9807024	0.9713314	0.962139	0.953119	0.944268
0.3	1.000039	0.96851	0.9389301	0.9111224	0.884931	0.860219	0.8368634
0.4	0.999751	0.930068	0.8695831	0.8165785	0.76974	0.728046	0.6906872
0.5	0.997393	0.875328	0.7801027	0.7037239	0.641087	0.588779	0.5444328
0.6	0.990813	0.809423	0.6842776	0.5927215	0.522828	0.467719	0.4231498
0.7	0.978815	0.739858	0.5943384	0.4964561	0.426134	0.373185	0.3318897
0.8	0.96151	0.67314	0.5168041	0.4188233	0.351728	0.302942	0.2658966
0.9	0.939967	0.613217	0.4532308	0.3584788	0.295922	0.251598	0.2185886
1	0.915694	0.561643	0.4024858	0.3122782	0.25433	0.214044	0.1844648
1.5	0.797388	0.410392	0.2714711	0.2004163	0.157488	0.128878	0.1085304
2	0.725583	0.355456	0.2298526	0.1671051	0.129728	0.10507	0.0876748



Thus, at $q = 0$ and $x = b^*$ we have

$$Rv2 = \frac{\int \sigma_y}{-qb} = \sum_{n=1}^{\infty} \left(\frac{+\frac{W_n}{q} \frac{(\pi n)^2}{(1+\Gamma)^2} + \frac{2}{\Gamma} \frac{\pi n}{(1+\Gamma)^2} \sin \alpha_n b - \frac{1}{\Gamma} 2k_n A \frac{C2}{0.5qb^2} \sin \alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A} \right) \sin \alpha_n b^* \ldots\ldots\ldots\ldots.128$$

Table 18 Rv 2 verses h/b for = 0.1, b = b*

h/b	A = 0	A = 5	A = 10	A = 15	A = 20	A = 25	A = 30
1E 13	0.999755	0.999755	0.9997549	0.9997549	0.999755	0.999755	0.9997549
0.1	0.985215	0.965852	0.9466779	0.9276897	0.908885	0.890262	0.8718173
0.2	0.966849	0.908325	0.85337	0.8016819	0.752991	0.707057	0.6636643
0.3	0.951827	0.861758	0.7846966	0.7180917	0.660015	0.608982	0.5638328
0.4	0.941455	0.829834	0.7444883	0.6772215	0.622917	0.578217	0.5408259
0.5	0.934249	0.80844	0.7225693	0.6601956	0.612811	0.575573	0.545519
0.6	0.92902	0.793924	0.7106718	0.6539237	0.612559	0.580927	0.5558527
0.7	0.925027	0.783905	0.703978	0.6519228	0.614939	0.587052	0.5650969
0.8	0.92186	0.776931	0.7000296	0.6514768	0.617498	0.592041	0.5720263
0.9	0.919299	0.77207	0.6976028	0.6515437	0.619588	0.595708	0.5769168
1	0.917218	0.768686	0.6960651	0.6517443	0.621145	0.598296	0.5802903
1.5	0.911498	0.762202	0.693493	0.6524328	0.624248	0.603175	0.586491
2	0.909658	0.761059	0.6931225	0.6525952	0.624776	0.60396	0.5874608

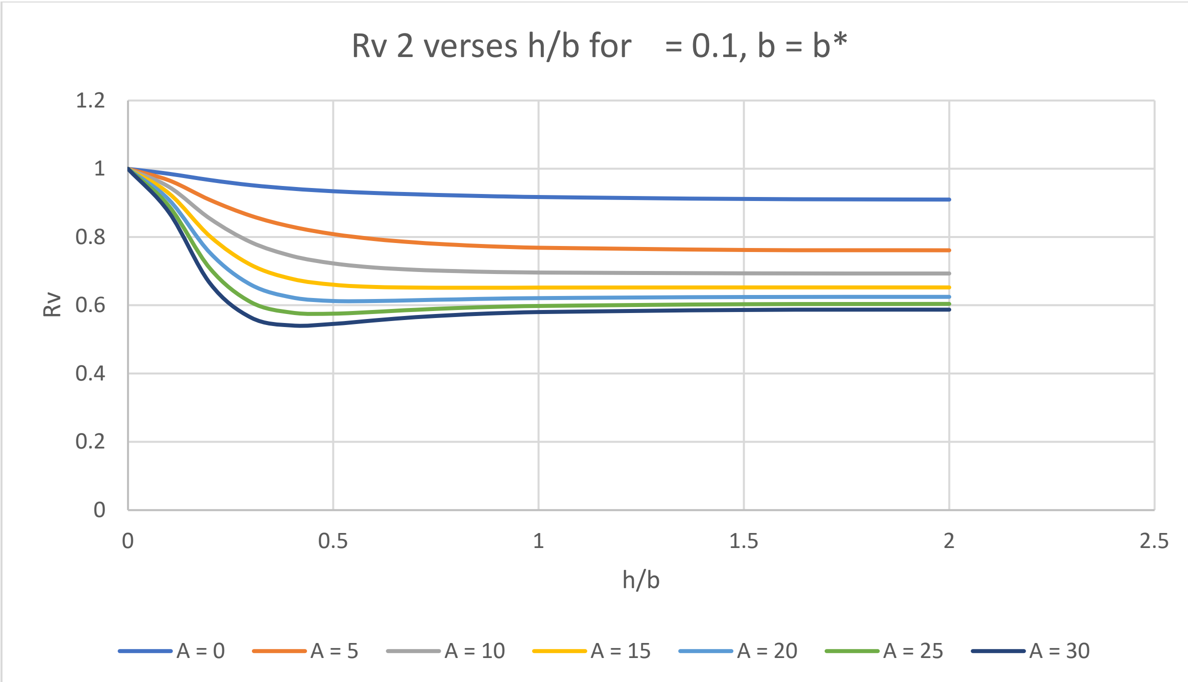
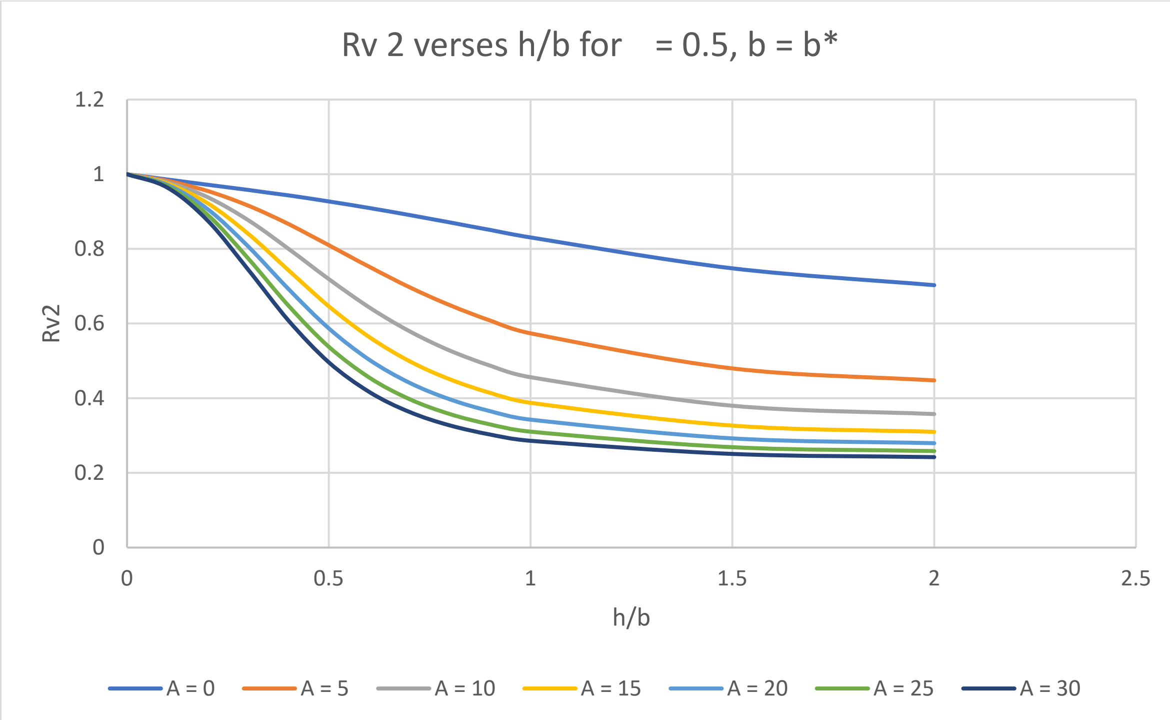


Table 19 Rv 2 verses h/b for = 0.5, b = b*

h/b	A = 0	A = 5	A = 10	A = 15	A = 20	A = 25	A = 30
1E 13	0.999909	0.999909	0.9999088	0.9999088	0.999909	0.999909	0.9999088
0.1	0.985886	0.982226	0.9785781	0.9749416	0.971317	0.967703	0.9641004
0.2	0.971833	0.954817	0.9381846	0.9219251	0.906027	0.890479	0.8752706
0.3	0.957787	0.91597	0.8769891	0.8405777	0.806502	0.774554	0.7445505
0.4	0.943192	0.866426	0.8002856	0.7427567	0.692299	0.647717	0.6080664
0.5	0.927158	0.809895	0.71888	0.646254	0.587005	0.537784	0.4962732
0.6	0.90952	0.751955	0.6432791	0.563794	0.50313	0.455309	0.4166402
0.7	0.890587	0.697399	0.5791212	0.4990945	0.441241	0.397398	0.362976
0.8	0.870787	0.649045	0.5275126	0.4504011	0.396893	0.357444	0.3270621
0.9	0.850617	0.607917	0.4871676	0.414316	0.365238	0.329719	0.3026859
1	0.830625	0.57386	0.4560401	0.387619	0.342476	0.310195	0.2857984
1.5	0.747966	0.479726	0.3798913	0.326481	0.292546	0.268689	0.250766
2	0.702596	0.447731	0.3573919	0.3097253	0.279535	0.258282	0.2422599



And the stress at $x = 0$ we have

$$\frac{\sigma_y}{q} = \frac{sft2}{q} = \sum_{n=1}^{\infty} 2k_n A \left(\frac{+\frac{W_n}{q} + \frac{2}{\Gamma}\frac{\sin \alpha_n b}{n\pi} + \frac{\pi n}{\Gamma(1+\Gamma)}\frac{C2}{0.5qb^2}\sin \alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A} \right) - \sum_{n=1}^{\infty}\left(\frac{W_n}{q} + \frac{2}{\Gamma}\frac{\sin \alpha_n b}{n\pi}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 129$$

And the reaction at $x = b + t$ we have

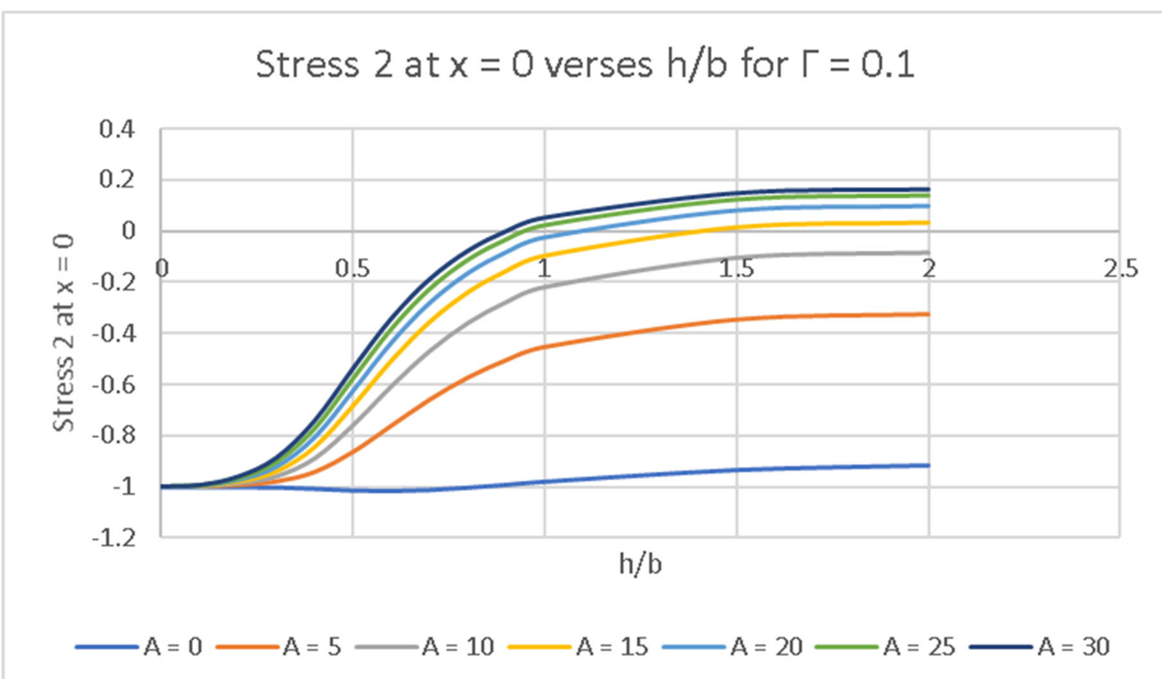
$$Reaction = \sum_{n=1}^{\infty} 2k_n A \left(\frac{\frac{W_n}{q} + \frac{2}{\Gamma}\frac{\sin \alpha_n b}{n\pi} + \frac{\pi n}{\Gamma(1+\Gamma)}\frac{C2}{0.5qb^2}\sin \alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A} \right) \cos \alpha_n n\pi - \sum_{n=1}^{\infty}\left(\frac{W_n}{q} + \frac{2}{\Gamma}\frac{\sin \alpha_n b}{n\pi}\right) \cos \alpha_n n\pi \ldots\ldots\ldots\ldots\ldots\ldots\ldots 130$$

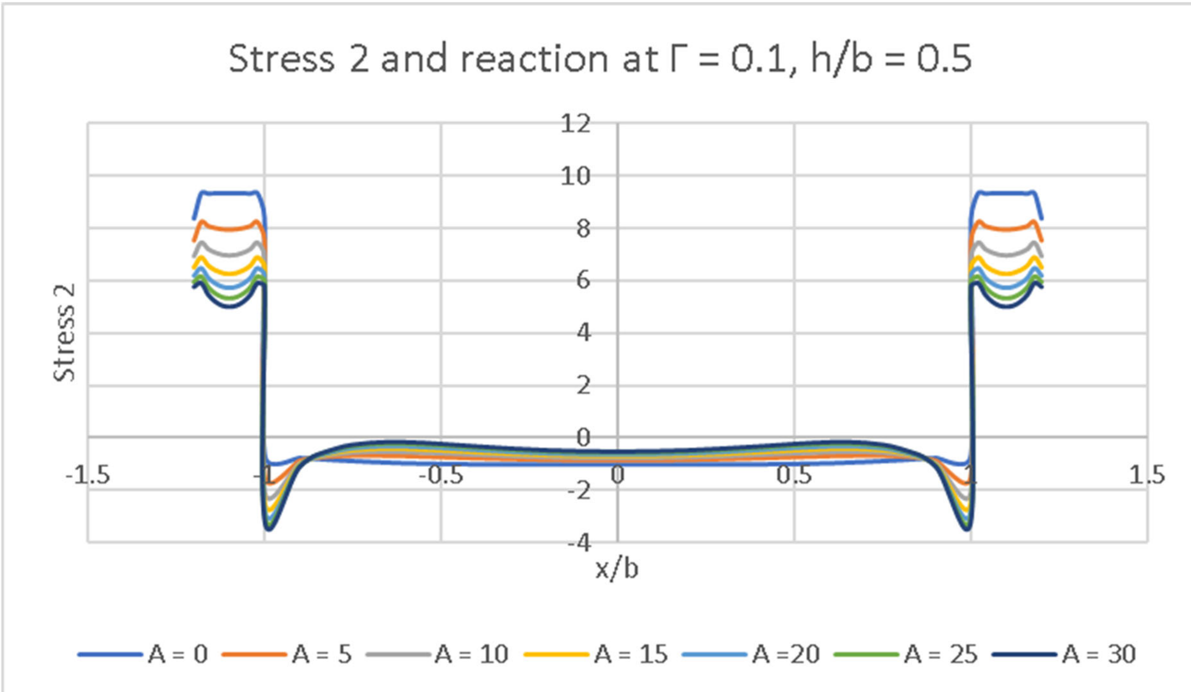
Please note the stress with wn is not define at $\pm b$ because it reverses from compression to tension for the reaction. So, for the shear and stress depending on how high n is taken for conversion calculations, the stress can at $x = \pm 0.999b$ and not b. Also, this happens for h/b is small. So, for example when $h/b = .001$ with nominal A the live load stress on the beam should match the dead

load stress on the beam at *x*/*b* =0.99. Attached on my web page is an excel spreadsheet. The best is to have a general program for the solution where *n* can be taken to a very high number. The spreadsheet goes to *n* = 5000 so it can be posted on my webpage for a smaller file size. It is recommended to extend to at least *n* = 20000.
Again, we assume the beam is glued to the mass media and it can experience a vertical stress and a tension stress.

Table 20 Stress 2 at x = 0 verses h/b for   = 0.1

h/b	A = 0	A = 5	A = 10	A = 15	A = 20	A = 25	A = 30
1E 13	0.99932	0.99932	0.99932	0.99932	0.99932	0.99932	0.999321
0.1	0.99938	0.99855	0.99772	0.996889	0.99606	0.99523	0.994408
0.2	0.99934	0.99279	0.98631	0.97991	0.97359	0.96734	0.961172
0.3	1.00008	0.97922	0.95932	0.94026	0.92199	0.90444	0.887544
0.4	1.00565	0.94167	0.88908	0.844545	0.80599	0.77203	0.741693
0.5	1.01221	0.86273	0.75919	0.682763	0.62371	0.57649	0.537687
0.6	1.01401	0.7596	0.60708	0.506251	0.43513	0.38258	0.342362
0.7	1.00978	0.65747	0.46918	0.354556	0.27903	0.22653	0.188635
0.8	1.00104	0.57045	0.35958	0.238664	0.16285	0.11256	0.077931
0.9	0.98992	0.50218	0.27816	0.155028	0.08053	0.03282	0.001195
1	0.97814	0.45104	0.21972	0.09628	0.02347	0.021962	0.0511994
1.5	0.93391	0.34371	0.10557	0.0148063	0.082425	0.122414	0.1464632
2	0.91639	0.32277	0.08579	0.0330934	0.099373	0.138209	0.1612637




Stress 2 and reaction at Γ = 0.1, h/b = 0.5
Stress 2
x/b
12
10
8
6
4
2
0
-2
-4
-1.5
-1
-0.5
0
0.5
1
1.5
A = 0
A = 5
A = 10
A = 15
A =20
A = 25
A = 30

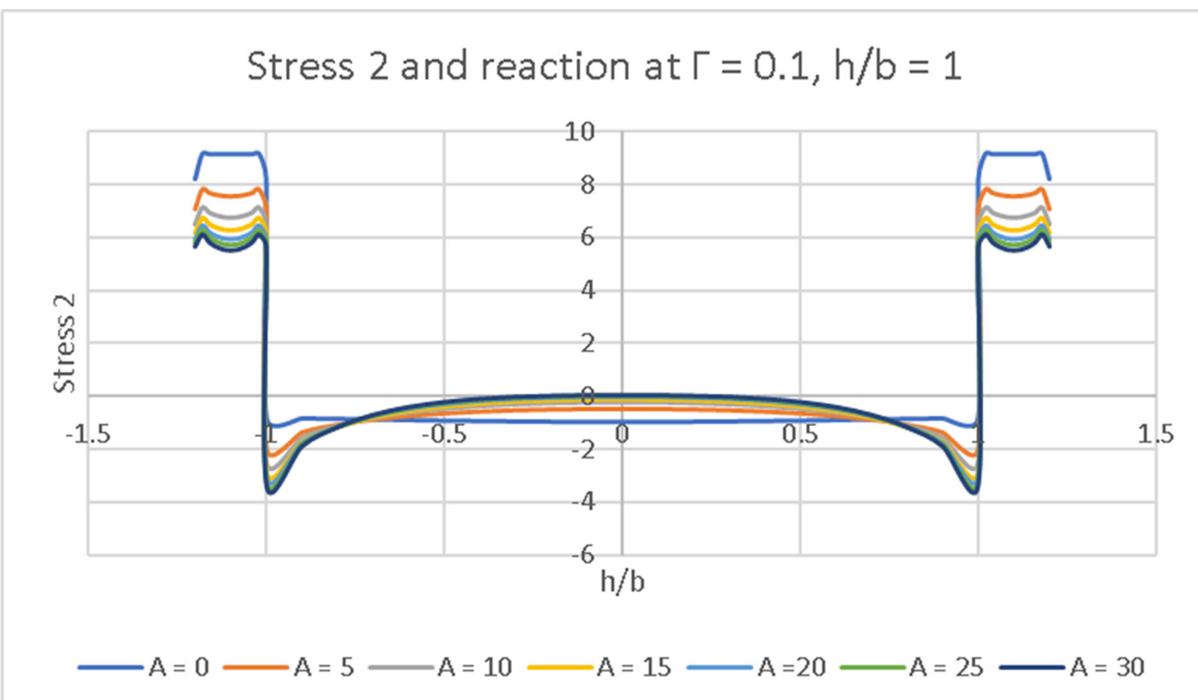
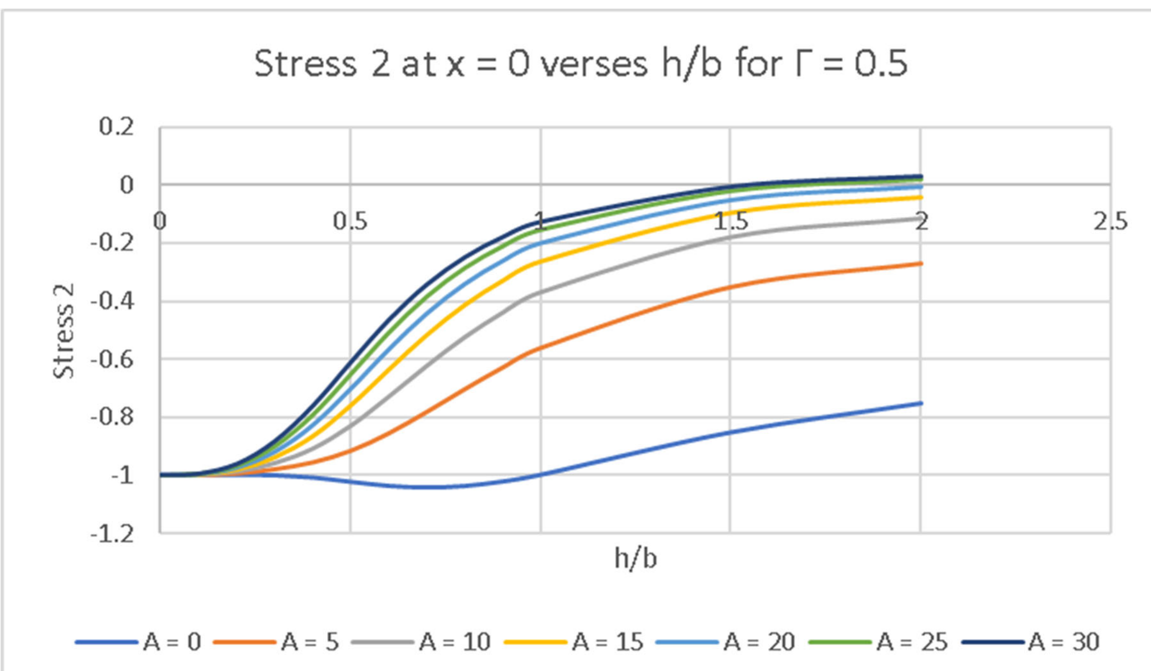
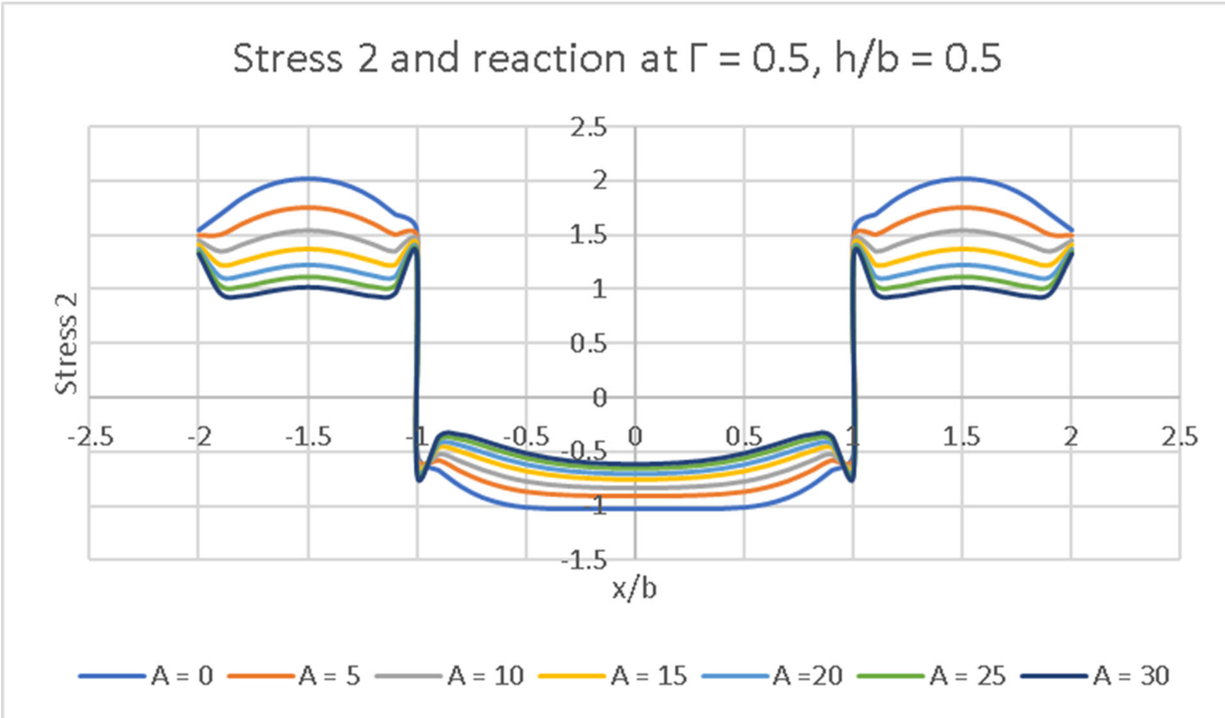
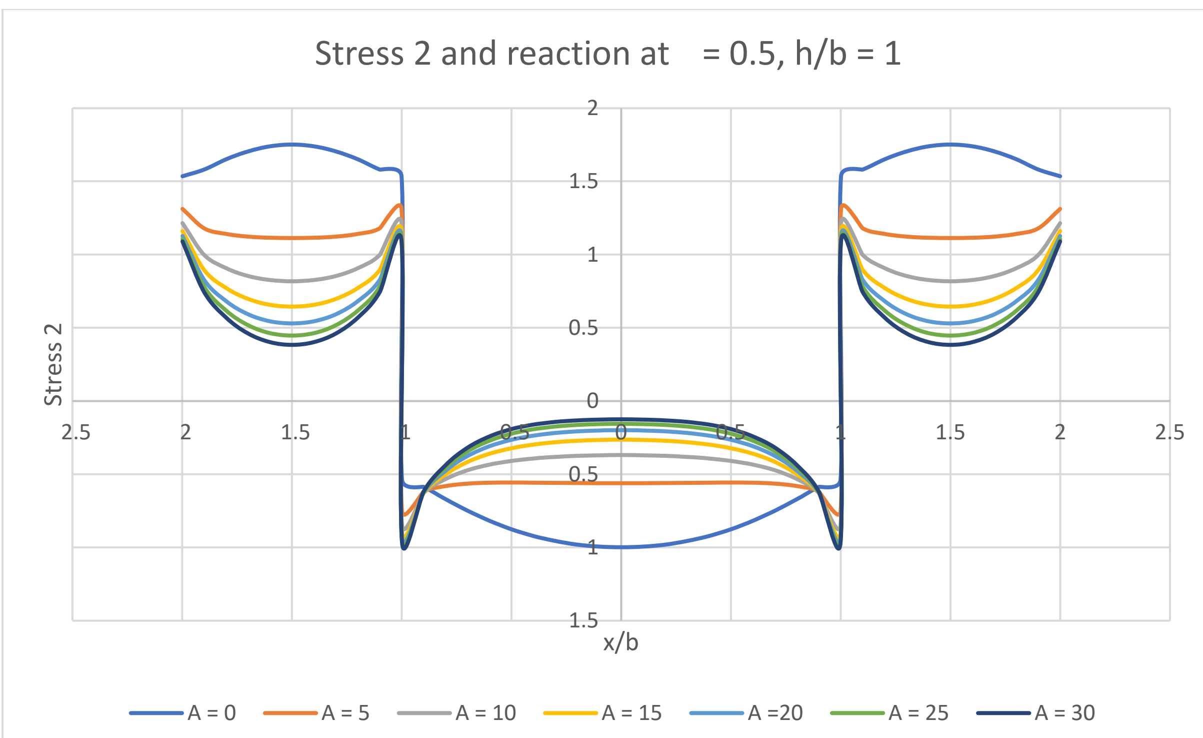

Stress 2 and reaction at Γ = 0.1, h/b = 1
Stress 2
h/b
10
8
6
4
2
0
-2
-4
-6
-1.5
-1
-0.5
0
0.5
1
1.5
A = 0
A = 5
A = 10
A = 15
A =20
A = 25
A = 30

Table 21 Stress 2 at x = 0 verses h/b for   = 0.5

h/b	A = 0	A = 5	A = 10	A = 15	A = 20	A = 25	A = 30
1E 13	0.99989	0.99989	0.99989	0.99989	0.99989	0.99989	0.99989
0.1	0.99993	0.999094	0.998263	0.997433	0.996604	0.99578	0.994951
0.2	0.99988	0.99327	0.986744	0.980302	0.973943	0.96767	0.961469
0.3	1.00049	0.978746	0.957953	0.938047	0.918969	0.90066	0.883083
0.4	1.00831	0.954998	0.907667	0.865283	0.827044	0.79232	0.760609
0.5	1.0234	0.914948	0.829169	0.759415	0.70143	0.65236	0.610211
0.6	1.03734	0.854884	0.728687	0.63611	0.565232	0.50918	0.463698
0.7	1.04297	0.780886	0.622444	0.516696	0.44133	0.38504	0.341485
0.8	1.03763	0.702642	0.523781	0.413591	0.339515	0.28668	0.247348
0.9	1.02208	0.627844	0.438962	0.329914	0.25993	0.21183	0.177128
1	0.99879	0.560712	0.369056	0.263956	0.198934	0.15556	0.125089
1.5	0.85212	0.351264	0.179459	0.096764	0.0504	0.02211	0.003957
2	0.75185	0.270081	0.115318	0.04363	0.004828	0.017936	0.0318659



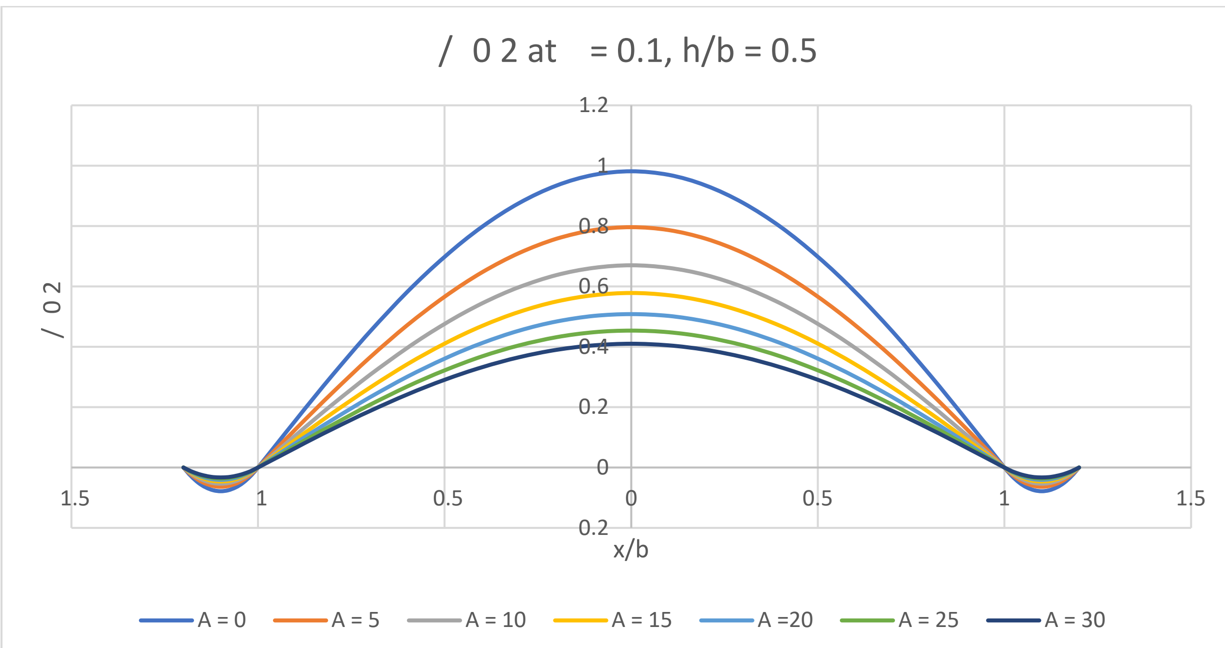

Stress 2 and reaction at Γ = 0.5, h/b = 0.5
Stress 2
x/b
A = 0
A = 5
A = 10
A = 15
A =20
A = 25
A = 30

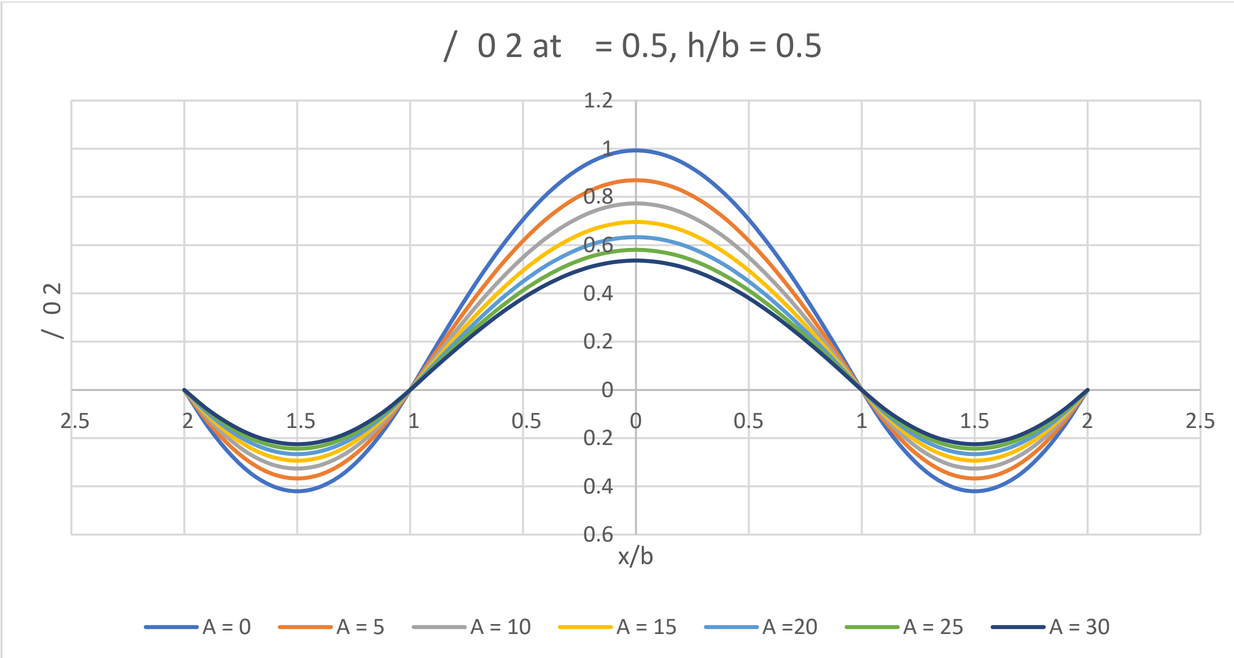

Stress 2 and reaction at = 0.5, h/b = 1
Stress 2
x/b
A = 0
A = 5
A = 10
A = 15
A =20
A = 25
A = 30

$$EIv_n = -\frac{\frac{W_n}{\alpha_n} + 2q\frac{b}{t(b+t)}\frac{\sin\alpha_n b}{\alpha_n^2} + \frac{2C2}{t}\sin\alpha_n b}{\alpha_n^3 + \frac{sk_n}{\beta EI}}$$

$$\frac{EI\ v(x)2}{-\frac{5qb^4}{24}} = \frac{1}{\frac{5}{24}}\sum_{n=1}^{\infty}\frac{\frac{W_n}{q}\frac{(1+\Gamma)}{\pi n} + 2\frac{(1+\Gamma)}{\Gamma}\frac{\sin\alpha_n b}{(\pi n)^2} + \frac{1}{\Gamma}\frac{C2}{0.5qb^2}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}(\cos\alpha_n x - \cos\alpha_n b) \ldots\ldots 131$$

The deflection does not take into account the moment of inertia of the elastic media, so it is conservative. But for a media such as soils where tension is not allowed it is accurate and useful.





$$\frac{EI\ v'(x)2}{\frac{qb^3}{3}} = 3\sum_{n=1}^{\infty} \frac{\frac{W_n}{q} + \frac{2}{\Gamma}\frac{\sin \alpha_n b}{\pi n} + \frac{n\pi}{\Gamma(1+\Gamma)}\frac{C2}{0.5qb^2}\sin \alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A} \sin \alpha_n x \ldots\ldots\ldots\ldots\ldots..132$$

We rewrite 2βkn from before as

$$2\beta k_n = -\frac{a_n + c_n}{\frac{d_n}{2\beta}}$$

$$\frac{d_n}{2\beta} = \frac{a_n - b_n - c_n - 1}{2}$$

And we have

$$W_n = -\frac{(a_n + c_n)(w_{n1} - w_{n2} - w_{n3})w_n}{a_n - b_n - c_n - 1} + (w_{n1} + w_{n3})w_n$$

We note ρ does not enter the equations

The total reaction at the column is $2(b + t)\,p + 2(b + t)\,q$.

Once D is found the stresses for $p = 0$ are

$$\sigma_y = -\sum_{n=\varphi}^{\infty} cos\,\alpha_n\,x\left[\left(a_n D + w_{n1}w_n + (b_n D + w_{n2}w_n)\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{\alpha_n y} + \left(c_n D + w_{n3}w_n + D\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y} + \frac{2q}{\Gamma}\frac{\sin\alpha_n b}{n\pi}\right] \quad -b < x < b \ldots\ldots 133$$

$$\sigma_x = k_0\sum_{n=\varphi}^{\infty} cos\,\alpha_n\,x\left[\left(a_n D + w_{n1}w_n + (b_n D + w_{n2}w_n)\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{\alpha_n y} + \left(c_n D + w_{n3}w_n + D\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y} - \frac{2q}{\Gamma}\frac{\sin\alpha_n b}{n\pi}\right] + 2k_0\sum_{n=\varphi}^{\infty} cos\,\alpha_n\,x[(b_n D + w_{n2}w_n)e^{\alpha_n y} - De^{-\alpha_n y}] \quad -b < x < b \ldots\ldots 134$$

$$\tau_{xy} = \sum_{n=\varphi}^{\infty} sin\,\alpha_n\,x\left[\left((a_n + b_n)D + w_{n1}w_n + w_{n2}w_n + (b_n D + w_{n2}w_n)\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{\alpha_n y} + \left(-(c_n - 1)D - w_{n3}w_n - D\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y}\right] \quad -b < x < b \ldots\ldots.. 135$$

$$D = -\frac{s\alpha_n v_n}{\beta d_n} - \frac{W1_n}{d_n} = -q\frac{2k_n A}{(a_n + c_n)}\left(\frac{+\frac{W_n}{q} + \frac{2}{\Gamma}\frac{\sin\alpha_n b}{n\pi} + \frac{\pi n}{\Gamma(1+\Gamma)}\frac{C2}{0.5qb^2}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}\right) - \frac{W1_n}{d_n} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. 136$$

Where:

$$W1_n = (w_{n1} - w_{n2} - w_{n3})w_n$$

$$a_n = -\left[c_n(1 + 2\alpha_n h) + 2\alpha_n^2 h^2\right]e^{-2\alpha_n h}$$

$$b_n = [2c_n - 1 + 2\alpha_n h]e^{-2\alpha_n h}$$

$$c_n = 1 + \frac{2\alpha_n^2 h^2 e^{-2\alpha_n h}}{(1 - 2\alpha_n h)e^{-2\alpha_n h} - 1}$$

$$d_n = a_n - b_n - c_n - 1$$

$$k_n = -\frac{a_n + c_n}{d_n}$$

$$W_n = -\frac{(a_n + c_n)(w_{n1} - w_{n2} - w_{n3})w_n}{(a_n - b_n - c_n - 1)} + (w_{n1} + w_{n3})w_n$$

$$w_{n1} = -w_{n3}(1 + 2\alpha_n h)e^{-2\alpha_n h} - (1 + \alpha_n h)e^{-\alpha_n h}$$

$$w_{n2} = 2w_{n3}e^{-2\alpha_n h} + e^{-\alpha_n h}$$

$$w_{n3} = \frac{\alpha_n h e^{-\alpha_n h}}{(1 - 2\alpha_n h)e^{-2\alpha_n h} - 1}$$

Rewrite for the excel sheet

$$\sigma_y = -\sum_{n=\varphi}^{\infty} cos\,\alpha_n\,x\left[\left(a_{n1}D + w_{n11}w_n + (b_{n1}D + w_{n21}w_n)\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n(h-y)} + \left(c_nD + w_{n3}w_n + D\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y} + \frac{2q}{\Gamma}\frac{\sin\alpha_n b}{n\pi}\right] \quad -b < x < b \ldots\ldots 137$$

$$\sigma_x = k_0\sum_{n=\varphi}^{\infty} cos\,\alpha_n\,x\left[\left(a_{n1}D + w_{n11}w_n + (b_{n1}D + w_{n21}w_n)\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n(h-y)} + \left(c_nD + w_{n3}w_n + D\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y} - \frac{2q}{\Gamma}\frac{\sin\alpha_n b}{n\pi}\right] + 2k_0\sum_{n=\varphi}^{\infty} cos\,\alpha_n\,x\left[(b_{n1}D + w_{n21}w_n)e^{-\alpha_n(h-y)} - De^{-\alpha_n y}\right] \quad -b < x < b \ldots 138$$

$$\tau_{xy} = \sum_{n=\varphi}^{\infty} sin\,\alpha_n\,x\left[\left((a_{n1} + b_{n1})D + w_{n11}w_n + w_{n21}w_n + (b_{n1}D + w_{n21}w_n)\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n(h-y)} + \left(-(c_n - 1)D - w_{n3}w_n - D\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y}\right] \quad -b < x < b \ldots\ldots 139$$

$$a_{n1} = a_n e^{\alpha_n y} = -\left[c_n(1 + 2\alpha_n h) + 2\alpha_n^2 h^2\right]e^{-\alpha_n h}$$

$$b_{n1} = b_n e^{\alpha_n y} = [2c_n - 1 + 2\alpha_n h]e^{-\alpha_n h}$$

$$w_{n11} = w_{n1}e^{\alpha_n y} = -w_{n3}(1 + 2\alpha_n h)e^{-\alpha_n h} - (1 + \alpha_n h)$$

$$w_{n21} = w_{n2}e^{\alpha_n y} = 2w_{n3}e^{-\alpha_n h} + 1$$

Example 3:

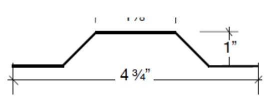
In example 1 we used a 30 ft wall to approximate an infinite wall. When redoing the calculation with h/b = 15 and A = 50.85 Rm is identical to the example = 0.084496 at x = 0.615b and Rv = 0.025824 verses 0.025824 at x = 0.3787b is also identical.

Example 4:

An underground tunnel made from timber 30 ft deep 8 ft wide. The ribs are 12x12 actual 11.25"x 11.25" timber spaces at 5 ft on center. It is required to investigate the 5 ft lagging spanning from rib to rib. It is a simple span condition. We try the lagging is 4x12 actual 3.5" x 11.25" $E_{s,eff}$=1111psi, $1/\beta = E_s/(1-\nu^2)$=1221 psi. t/b = 0.5. p = 120*30*11.25/12 = 3375 lbs/ft. K_0 = 0.428571 for = 0.3. x 1 /p = 0.23174 at x = 0 and y = 0 so we have compression everywhere and the solution is applicable for soils. A = 2.56303 and h/b =12. Thus Rm1 = 0.649475 thus Mmax =3375*0.649475*5^2/8 = 6849.93 lbs ft. Sx required for Fb = 900*1.1 = 990 psi for flatwise condition for 4x12 timber is 6849.93*12/990 = 83.03 in^3 > 11.25*3.5^2/6 = 22.97 in^3 so it is no good. We try 10x12 lagging x 1 /p = 0.41282 at x = 0 and y = 0 so we have compression everywhere and the solution is applicable for soils. A = 0.13885 and h/b =12. Thus Rm1 = 0.972179 thus Mmax =3375*0.972179*5^2/8 = 10253.45 lbs ft. Sx required for Fb = 900*1.1 = 990 psi for flatwise condition for 10x12 timber is 10253.45*12/990 = 124.28 in^3 < 11.25*9.25^2/6 = 160.43 in^3. At x/b = 0.69167 Rv = 0.672171 or V = 1.5*3375*0.672171*2.5/(11.25*9.25) = 84.12 psi < 95 psi so the shear is OK. Thus, 10x12 lagging is good. Finding the strength of the ribs is not part of this example.

Example 5:

A semicircular culvert measures 15 ft to the above grade at the center of the semicircle. The radius is 10 ft so at the bottom it is 20 width. Soil weight is 120 pcf K_0 = 0.5. We assume 7 ft lateral for the horizontal pressure and 14.28 ft vertical pressure with 15+3 = 18 ft overburden. The culver has ribs made from bended pipe at 2.4 ft O.C. and the lagging is corrugated sheet metal. The 2.4 ft was selected to have everything in compression in the soil. We are to check the strength of the corrugated sheet metal. We assume the corrugated sheet metal is simply supported on the ribs.



SECTION PROPERTIES									
				TOP IN COMPRESSION			BOTTOM IN COMPRESSION		
GAUGE	NOM. THICK (IN.)	WT. (PLF)	Fy (KSI)	I_x (in.^{4})	S_x (in.^{3})	F_b (KSI)	I_x (in.^{4})	S_x (in.^{3})	F_b (KSI)
18	.049	0.97	33.0	.0471	.0949	19.8	.0471	.0853	19.8

So Es,eff =1111psi, $1/\beta = Es/(1-v^2)$=1221 psi for = 0.3. I = .0471 *b* = 1.2 ft Es = 29000 ksi thus *A* = 16.02, t/b = 0.5 and s = 4.75" *p* = 120*18 = 2160 lbs/ft. Rm1 = 0.210373 Rv1 = 0.441536 thus Max = 2.160*0.210373*2.4^2/8 = 0.3272 kip ft. The allowable section modulus = 0.3272*12/19.8 = 0.918 in^3 per 12 inch strip for s = 4.75 the available section modulus becomes 0.0853*12/4.75 = 0.21 in^3 > 0.918 in^3 so the flexure is OK. The shear V = 2.160*0.441536*2.4/2 = 1.1445 Kips.
The area = 0.97/490 = 0.002 ft^2 or 0.285 in^2 so fv = 1.1445/0.285 = 4.01 ksi < 0.4*33 = 13.2 ksi OK. Checking the lateral pressure, we have at the bottom (15+7) *120*K_0 = 1320 lbs/ft < 2160. Thus, the vertical pressure controls the design of the corrugated sheet metal. Thus, the corrugated sheet metal checks and is OK. The design of the ribs made out of pipes is not part of the example.

<u>Closed form solution #4 fixed span beam with height *h*:</u>

Setting the rotation to be zero at $x = \pm b$ for $q = 0$ we have.

$$0 = \frac{3}{\Gamma}\sum_{n=1}^{\infty}\frac{2(1+\Gamma)\dfrac{\sin\alpha_n b}{\pi n} + \dfrac{C2}{0.5pb^2}\dfrac{\pi n}{(1+\Gamma)}\sin\alpha_n b}{\dfrac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}\sin\alpha_n b$$

$$0 = \sum_{n=1}^{\infty}\frac{2(1+\Gamma)\dfrac{\sin\alpha_n b}{\pi n}}{\dfrac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}\sin\alpha_n b + \sum_{n=1}^{\infty}\frac{\dfrac{C2}{0.5pb^2}\dfrac{\pi n}{(1+\Gamma)}\sin\alpha_n b\sin\alpha_n b}{\dfrac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}$$

$$\frac{C2\;1}{0.5pb^2} = -\frac{\sum_{n=1}^{\infty}\dfrac{2(1+\Gamma)\dfrac{(\sin\alpha_n b)^2}{\pi n}}{\dfrac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}}{\sum_{n=1}^{\infty}\dfrac{\dfrac{\pi n}{(1+\Gamma)}(\sin\alpha_n b)^2}{\dfrac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..140$$

And *Rm1+* is, note: the maximum positive moment may not be at *x* = 0 depending on *A*

$$Rm1+= \frac{M+}{\dfrac{pb^2}{6}} = 3\left[\frac{1}{\Gamma}\sum_{n=1}^{\infty}\frac{4\sin\alpha_n b - 4k_n A\left(\dfrac{C2}{0.5pb^2}\right)\dfrac{(1+\Gamma)}{\pi n}\sin\alpha_n b}{\dfrac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A} + \frac{C2}{0.5pb^2}\right]\ldots\ldots..141$$

Table 22 Rm 1+ verses h/b for   = 0.1

h/b	A = 0	A = 10	A = 20	A = 30	A = 40	A = 50	A = 60
1E 13	0.999951	0.999951	0.999951	0.999951	0.99995096	0.999951	0.999951
0.1	0.999951	0.998383	0.996821	0.995263	0.99371005	0.992162	0.9906192
0.2	0.999951	0.988358	0.977034	0.965971	0.95515929	0.94459	0.9342565
0.3	0.999951	0.964717	0.93186	0.901151	0.8723885	0.845396	0.8200173
0.4	0.999951	0.928113	0.86552	0.81054	0.76189838	0.718586	0.6797946
0.5	0.999951	0.884514	0.79151	0.715142	0.651433	0.597565	0.5514872
0.6	0.999951	0.841114	0.722643	0.631228	0.55878815	0.50014	0.4518116
0.7	0.999951	0.803077	0.665842	0.565243	0.48869295	0.428736	0.3806792
0.8	0.999951	0.772618	0.622556	0.516785	0.43867458	0.378937	0.3319877
0.9	0.999951	0.749726	0.591226	0.48265	0.40415017	0.345104	0.2993234
1	0.999951	0.73327	0.569312	0.459225	0.38078797	0.322454	0.2776398
1.5	0.999951	0.70313	0.530431	0.418548	0.34084387	0.284176	0.2413245
2	0.999951	0.699571	0.525944	0.413925	0.33635337	0.279908	0.2373007

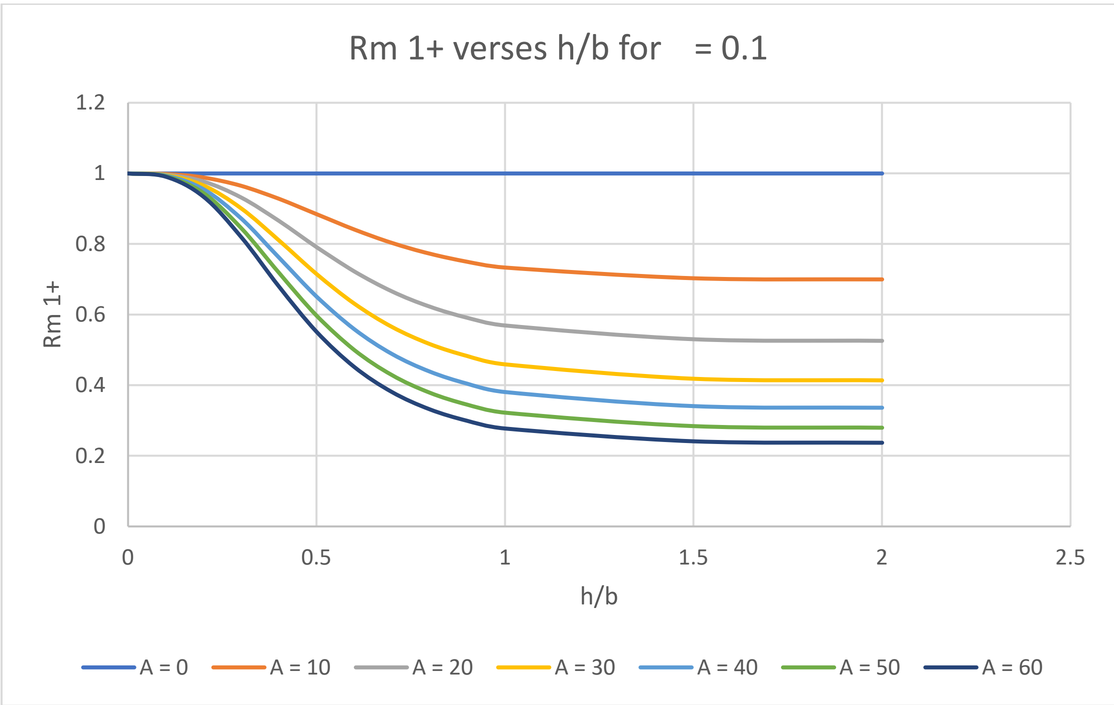
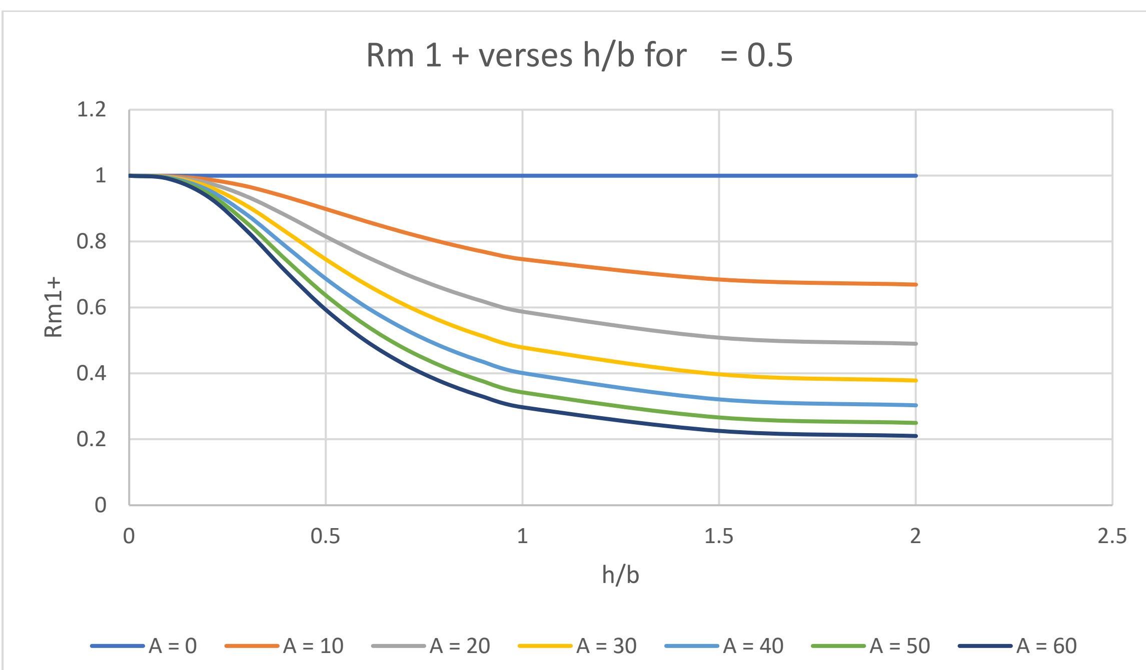


Table 23 Rm 1 + verses h/b for = 0.5

h/b	A = 0	A = 10	A = 20	A = 30	A = 40	A = 50	A = 60
1E 13	0.999909	0.999909	0.999909	0.999909	0.99990882	0.999909	0.9999088
0.1	0.999909	0.998372	0.996839	0.995312	0.99378887	0.992271	0.9907579
0.2	0.999909	0.988781	0.977913	0.967294	0.95691684	0.946773	0.9368535
0.3	0.999909	0.967093	0.936445	0.90776	0.88085563	0.855572	0.8317669
0.4	0.999909	0.935255	0.878464	0.828206	0.78343392	0.74331	0.7071564
0.5	0.999909	0.898799	0.815667	0.746202	0.68735739	0.636924	0.5932582
0.6	0.999909	0.862153	0.755944	0.671778	0.60359578	0.547353	0.5002502
0.7	0.999909	0.827676	0.702616	0.608077	0.53436923	0.475479	0.4274808
0.8	0.999909	0.796564	0.656754	0.555309	0.47872612	0.419123	0.3715985
0.9	0.999909	0.769451	0.618469	0.512661	0.43487749	0.375611	0.3291725
1	0.999909	0.746567	0.587329	0.478896	0.40087229	0.342413	0.2972326
1.5	0.999909	0.684949	0.508518	0.39703	0.32099004	0.266306	0.2254128
2	0.999909	0.669306	0.48961	0.378128	0.30305216	0.249574	0.2098857



And *Rm1-* is

$$Rm1 - = \frac{M -}{\frac{-pb^2}{3}} = -\frac{3}{2}\left[\frac{1}{\Gamma}\sum_{n=1}^{\infty}\frac{4\sin\alpha_n b - 4k_n A\left(\frac{C2}{0.5pb^2}\right)\frac{(1+\Gamma)}{\pi n}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}\cos\alpha_n b + \frac{C2}{0.5pb^2}\right] \ldots\ldots 142$$

Table 24 Rm 1 verses h/b for = 0.1

h/b	A = 0	A = 10	A = 20	A = 30	A = 40	A = 50	A = 60
1E 13	1.000025	1.000025	1.000025	1.000025	1.00002452	1.000025	1.0000245
0.1	1.000025	0.999284	0.998544	0.997807	0.99707129	0.996337	0.9956053
0.2	1.000025	0.994063	0.988203	0.982443	0.97678025	0.971211	0.9657331
0.3	1.000025	0.98062	0.962237	0.944789	0.92819977	0.912402	0.8973358
0.4	1.000025	0.95914	0.922618	0.889749	0.85997236	0.832841	0.8079907
0.5	1.000025	0.933217	0.877679	0.830639	0.7901768	0.754921	0.7238644
0.6	1.000025	0.907266	0.835588	0.778258	0.73115992	0.691636	0.6578891
0.7	1.000025	0.884476	0.800808	0.737005	0.68645644	0.64522	0.610796
0.8	1.000025	0.866215	0.774303	0.706731	0.65460182	0.61292	0.5786579
0.9	1.000025	0.85249	0.755128	0.685429	0.63265586	0.591034	0.5571726
1	1.000025	0.842624	0.741723	0.670825	0.61782886	0.576414	0.5429501
1.5	1.000025	0.824557	0.717955	0.645497	0.59252415	0.551767	0.5192063
2	1.000025	0.822424	0.715213	0.642621	0.5896826	0.549023	0.5165806

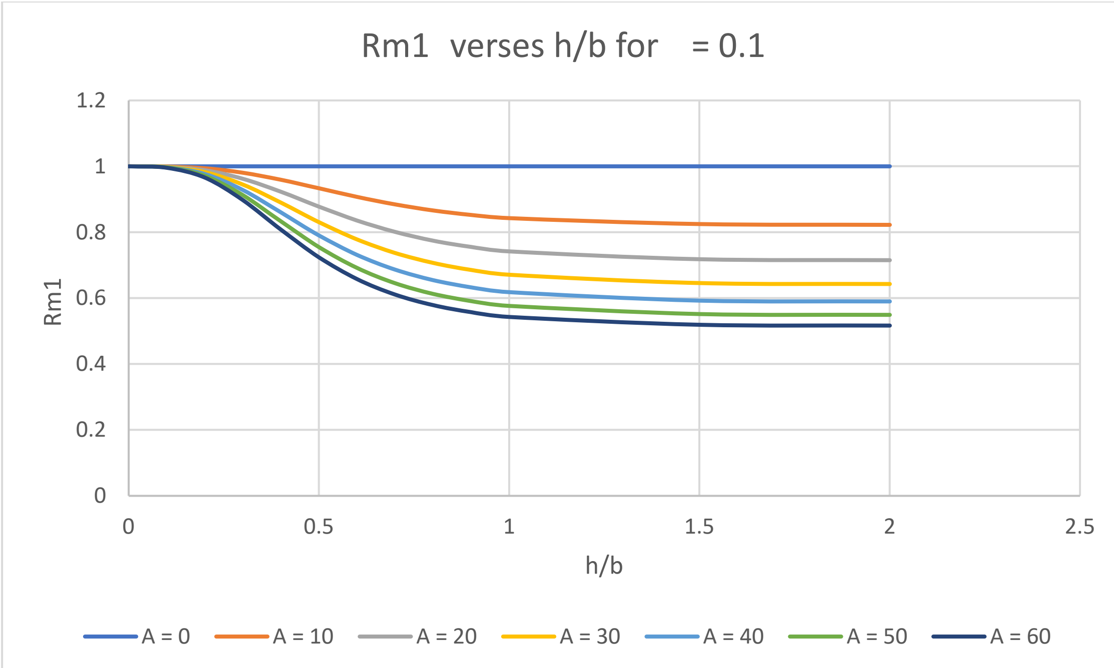
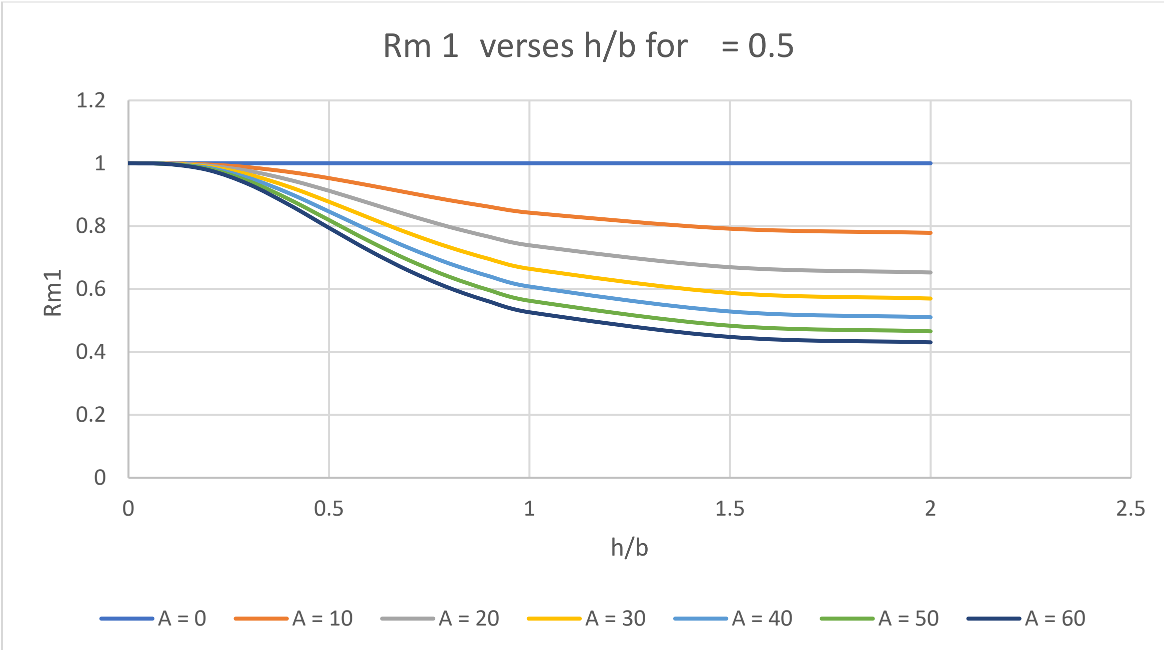


Table 25 Rm 1 verses h/b for = 0.5

h/b	A = 0	A = 10	A = 20	A = 30	A = 40	A = 50	A = 60
1E 13	1.000046	1.000046	1.000046	1.000046	1.00004559	1.000046	1.0000456
0.1	1.000046	0.999581	0.999118	0.998657	0.99819622	0.997737	0.9972794
0.2	1.000046	0.996195	0.99242	0.988718	0.98508605	0.981523	0.978026
0.3	1.000046	0.987408	0.97545	0.96411	0.95333555	0.943078	0.9332962
0.4	1.000046	0.97257	0.947801	0.92531	0.90475491	0.885861	0.8684047
0.5	1.000046	0.952792	0.912461	0.877482	0.84673991	0.819417	0.7949
0.6	1.000046	0.929925	0.87337	0.826488	0.78677611	0.752547	0.7226189
0.7	1.000046	0.905967	0.834211	0.77721	0.73052288	0.691362	0.6578864
0.8	1.000046	0.88272	0.79783	0.732948	0.68135135	0.639071	0.6036074
0.9	1.000046	0.861518	0.765953	0.69532	0.64052962	0.596489	0.5601135
1	1.000046	0.84312	0.739251	0.664601	0.60785843	0.562945	0.5262956
1.5	1.000046	0.792188	0.66965	0.587817	0.52869721	0.483616	0.4478665
2	1.000046	0.779074	0.652694	0.569804	0.51063227	0.465894	0.4306407



Rv1, the stress and deflections are the same just C2 is different

$$Rv1 = \frac{\int \sigma_y}{-pb} = \sum_{n=1}^{\infty} \left(\frac{\frac{2\pi n}{\Gamma(1+\Gamma)} - 2k_n A \frac{C2}{\Gamma 0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A} \right) \sin \alpha_n b \sin \alpha_n b^* \dots\dots\dots\dots\dots\dots\dots .143$$

Table 26 Rv 1 verses h/b for = 0.1

h/b	A = 0	A = 10	A = 20	A = 30	A = 40	A = 50	A = 60
1E 13	0.999755	0.999755	0.999755	0.999755	0.99975487	0.999755	0.9997549
0.1	0.999755	1.002742	1.005721	1.008692	1.01165495	1.01461	1.0175573
0.2	0.999755	1.005983	1.012076	1.018038	1.02387243	1.029584	1.0351756
0.3	0.999755	1.004562	1.008983	1.013055	1.01680963	1.020275	1.0234777
0.4	0.999755	0.999488	0.998921	0.998122	0.99713943	0.996013	0.9947744
0.5	0.999755	0.992637	0.98616	0.980203	0.9746773	0.969517	0.9646723
0.6	0.999755	0.985595	0.973884	0.963892	0.95516971	0.947419	0.9404372
0.7	0.999755	0.979366	0.963671	0.950967	0.94030811	0.931126	0.923056
0.8	0.999755	0.974366	0.955879	0.941476	0.92971965	0.919797	0.9112109
0.9	0.999755	0.970606	0.950243	0.934804	0.92243584	0.912137	0.9033131
1	0.999755	0.967903	0.946305	0.930234	0.91752207	0.90703	0.8980973
1.5	0.999755	0.962955	0.939327	0.922319	0.90915088	0.89844	0.8894139
2	0.999755	0.96237	0.938523	0.92142	0.90821191	0.897485	0.8884553

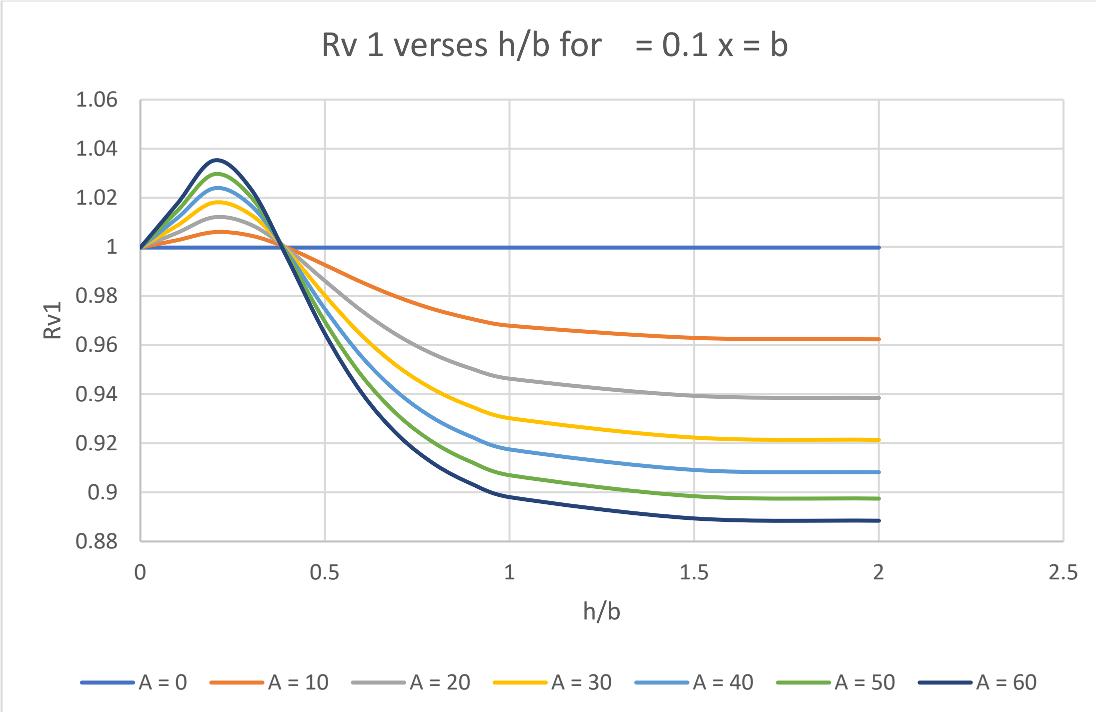
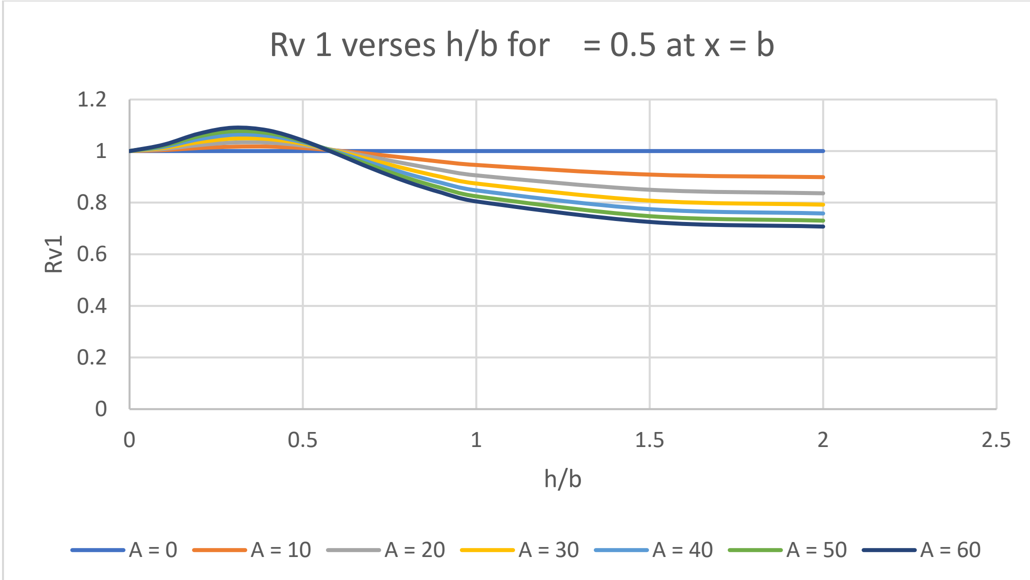


Table 27 Rv 1 verses h/b for = 0.5

h/b	A = 0	A = 10	A = 20	A = 30	A = 40	A = 50	A = 60
1E 13	0.999909	0.999909	0.999909	0.999909	0.99990883	0.999909	0.9999088
0.1	0.999909	1.004158	1.008399	1.012631	1.01685509	1.02107	1.0252767
0.2	0.999909	1.011678	1.023267	1.034681	1.04592511	1.057003	1.0679183
0.3	0.999909	1.016911	1.033074	1.048467	1.06314793	1.077171	1.090583
0.4	0.999909	1.017324	1.032742	1.046481	1.05879199	1.069878	1.0799054
0.5	0.999909	1.012099	1.021396	1.028516	1.03396112	1.038097	1.0411934
0.6	0.999909	1.0018	1.001239	0.999091	0.99589769	0.992008	0.9876571
0.7	0.999909	0.988184	0.976304	0.964597	0.95322172	0.942249	0.9317086
0.8	0.999909	0.973319	0.950515	0.930416	0.91238486	0.896008	0.8809992
0.9	0.999909	0.958879	0.92658	0.899784	0.87679877	0.856632	0.8386487
1	0.999909	0.945903	0.905877	0.874031	0.84754037	0.824835	0.8049596
1.5	0.999909	0.908771	0.85026	0.807871	0.77485945	0.747915	0.7251991
2	0.999909	0.899056	0.836507	0.792136	0.75806586	0.730537	0.7075026



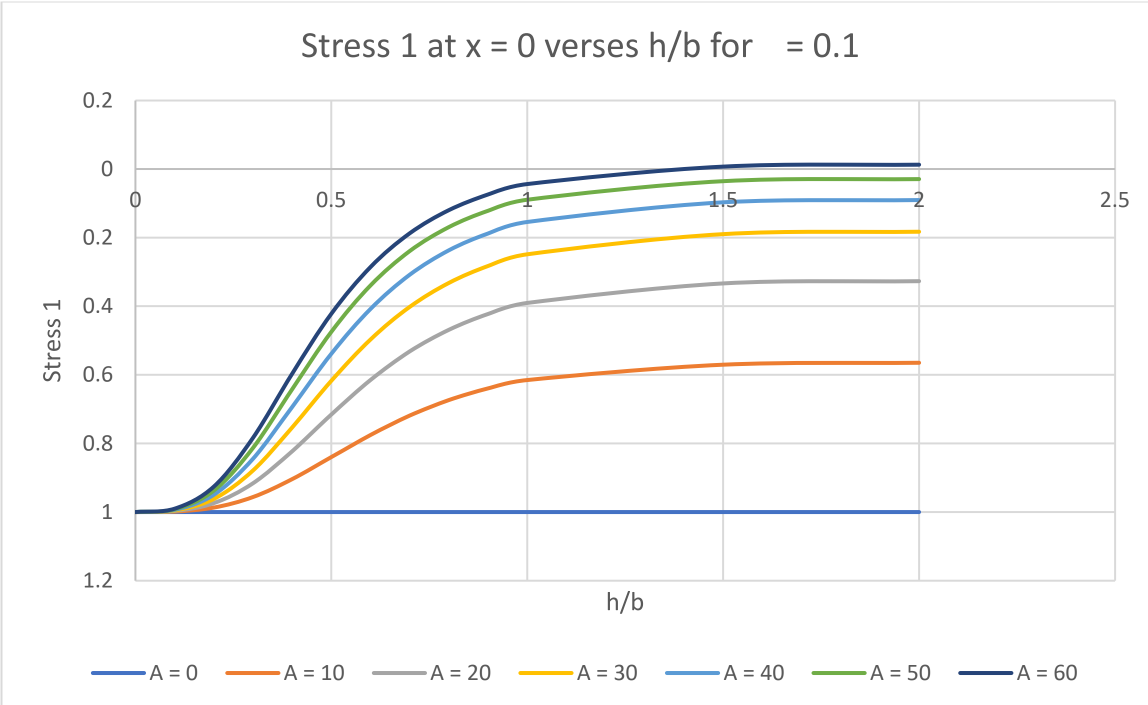
$$\frac{\sigma_y}{p} at\ x = 0 = \frac{sft1}{p} = \frac{1}{\Gamma}\sum_{n=1}^{\infty} 2k_n A \frac{2\frac{(1+\Gamma)}{\pi n} + \frac{\pi n}{(1+\Gamma)}\frac{C2}{0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A} \sin\alpha_n b - 1 \ldots\ldots\ldots 144$$

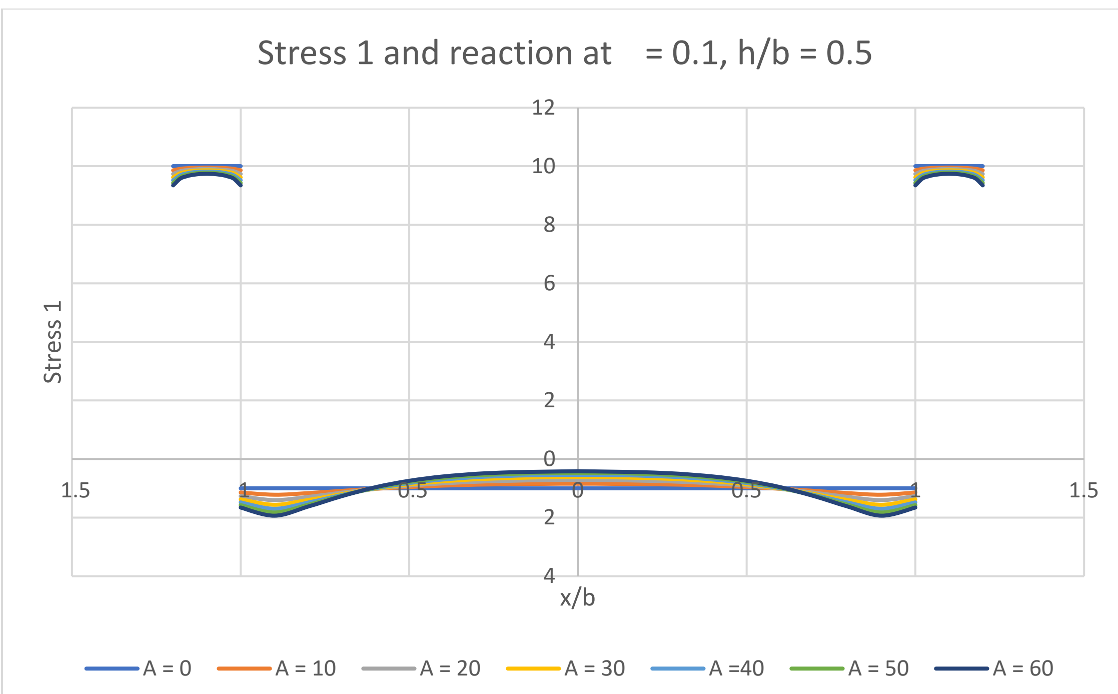
$$Reaction\ at\ x = b + t = \frac{sft1}{p}$$

$$= \frac{1}{\Gamma}\sum_{n=1}^{\infty} 2k_n A \frac{2\frac{(1+\Gamma)}{\pi n} + \frac{\pi n}{(1+\Gamma)}\frac{C2}{0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A} \sin\alpha_n b \cos\pi n + \frac{1}{\Gamma} \ldots\ldots\ldots\ldots 145$$

Table 28 Stress 1 at x = 0 verses h/b for
= 0.1

h/b	A = 0	A = 10	A = 20	A = 30	A = 40	A = 50	A = 60
1E 13	1	1	1	1	1	1	1
0.1	1	0.99834	0.99668	0.99502	0.9933774	0.99174	0.990099
0.2	1	0.98683	0.974	0.9615	0.9493263	0.93745	0.925877
0.3	1	0.95618	0.91592	0.87882	0.8445555	0.81282	0.783373
0.4	1	0.90391	0.82254	0.75306	0.6932617	0.64145	0.596278
0.5	1	0.83966	0.71528	0.61704	0.5382516	0.47424	0.421654
0.6	1	0.77527	0.61483	0.49664	0.4074426	0.33882	0.285181
0.7	1	0.71877	0.53181	0.4017	0.3081012	0.23908	0.18721
0.8	1	0.67351	0.46846	0.33181	0.2369526	0.16917	0.119707
0.9	1	0.63948	0.42256	0.28247	0.187685	0.12147	0.07416
1	1	0.61502	0.39043	0.24856	0.1542644	0.08942	0.04379
1.5	1	0.5702	0.33337	0.18957	0.0969685	0.03506	0.0073187
2	1	0.56491	0.32678	0.18286	0.0905168	0.02898	0.0129983


Stress 1 at x = 0 verses h/b for = 0.1
0.2
0
0.2
0.4
0.6
0.8
1
1.2
Stress 1
0
0.5
1
1.5
2
2.5
h/b
A = 0
A = 10
A = 20
A = 30
A = 40
A = 50
A = 60


Stress 1 and reaction at = 0.1, h/b = 0.5
12
10
8
6
4
2
0
2
4
Stress 1
1.5
1
0.5
0
0.5
1
1.5
x/b
A = 0
A = 10
A = 20
A = 30
A =40
A = 50
A = 60

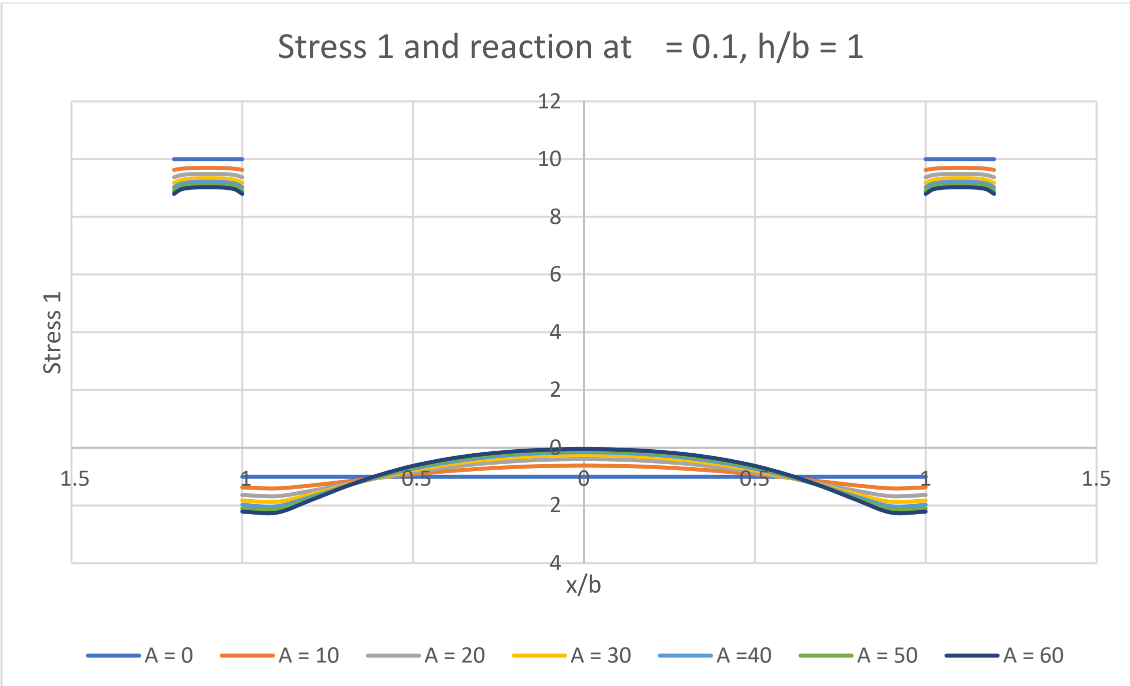
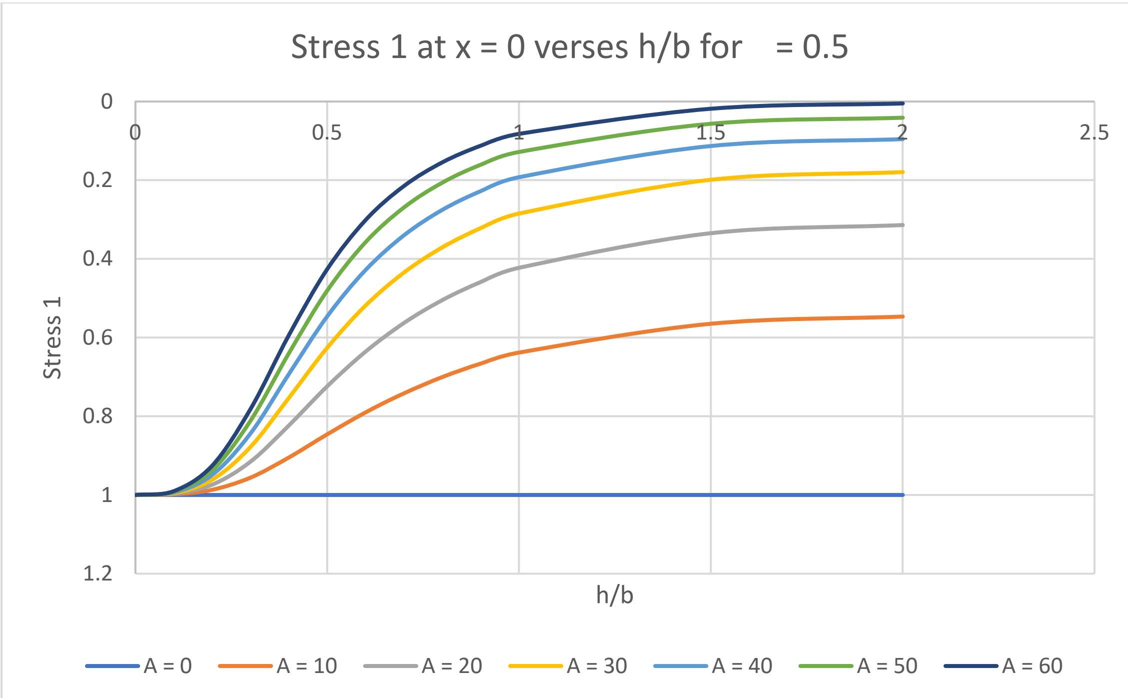
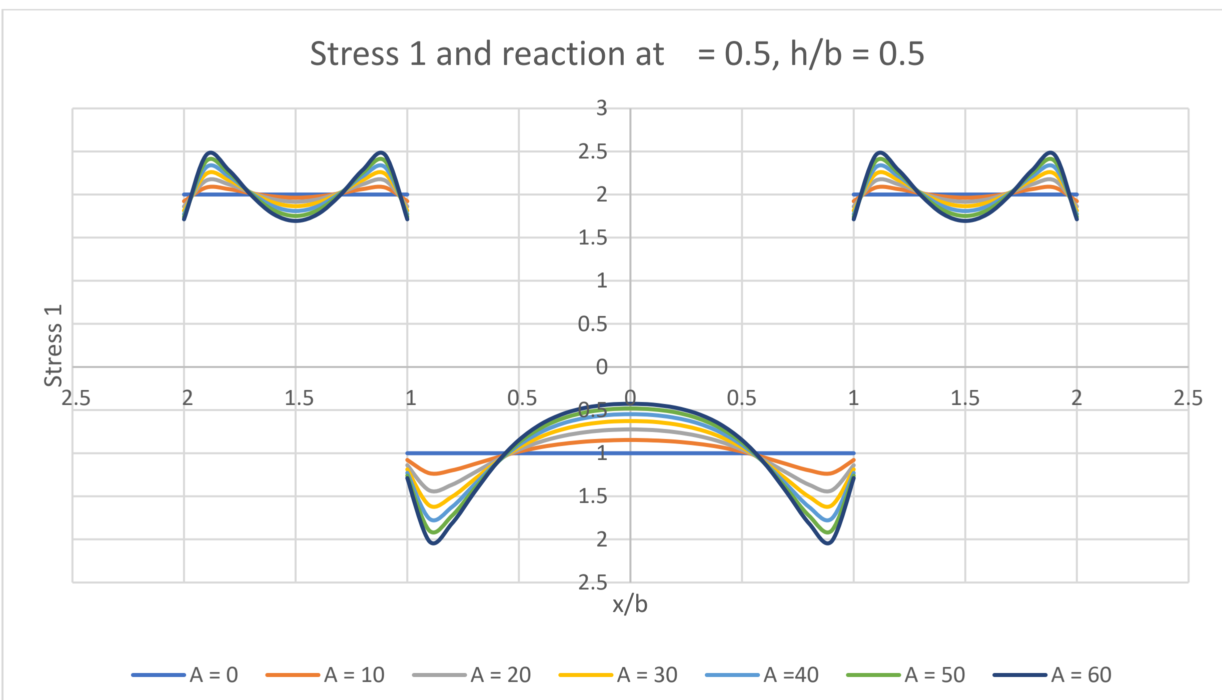


Table 29 Stress 1 at x = 0 verses h/b for = 0.5

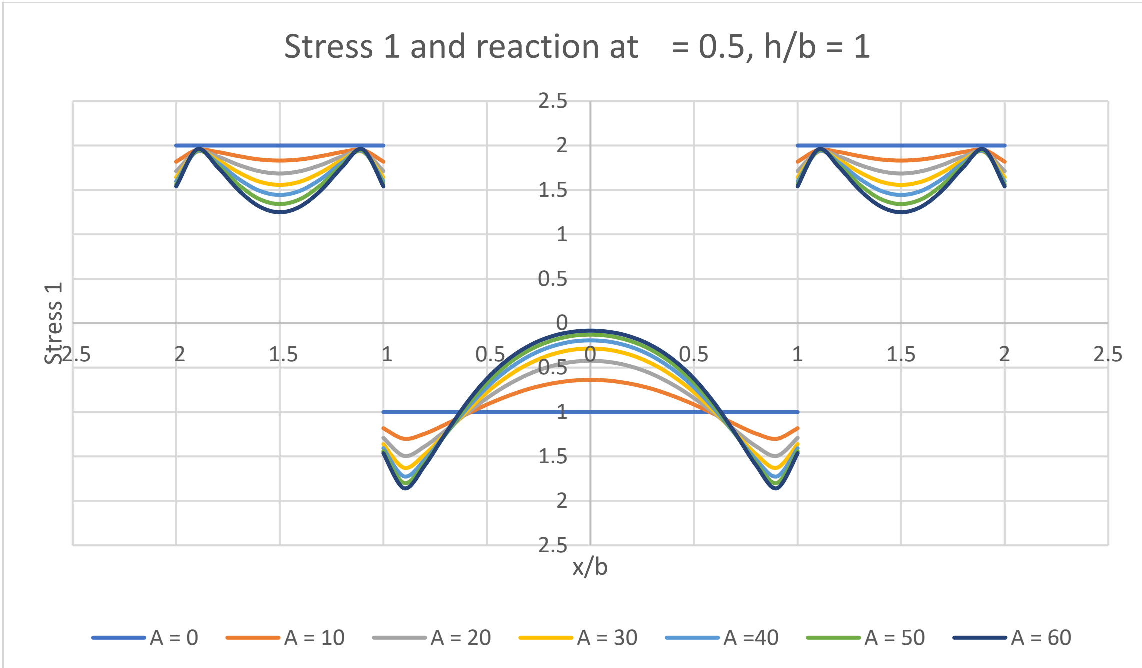
h/b	A = 0	A = 10	A = 20	A = 30	A = 40	A = 50	A = 60
1E 13	1	1	1	1	1	1	1
0.1	1	0.99834	0.99668	0.99502	0.9933774	0.99174	0.990099
0.2	1	0.98682	0.97399	0.96149	0.9493129	0.93744	0.92586
0.3	1	0.95598	0.91549	0.87816	0.8436601	0.8117	0.782032
0.4	1	0.90474	0.82343	0.75351	0.6929952	0.64031	0.594181
0.5	1	0.84591	0.72394	0.62593	0.5461756	0.48058	0.426126
0.6	1	0.79022	0.63551	0.51852	0.4283021	0.3576	0.301461
0.7	1	0.74158	0.56288	0.43471	0.3402506	0.26914	0.214713
0.8	1	0.70038	0.50477	0.37068	0.2755015	0.20617	0.15468
0.9	1	0.66609	0.45877	0.32193	0.2277433	0.16095	0.112522
1	1	0.63802	0.42266	0.28486	0.1923269	0.12809	0.082419
1.5	1	0.56486	0.33469	0.19877	0.1130208	0.05658	0.018398
2	1	0.5466	0.31402	0.17937	0.0956799	0.0413	0.004973



Stress 1 at x = 0 verses h/b for = 0.5
Stress 1
h/b
0
0.2
0.4
0.6
0.8
1
1.2
0
0.5
1
1.5
2
2.5
A = 0
A = 10
A = 20
A = 30
A = 40
A = 50
A = 60

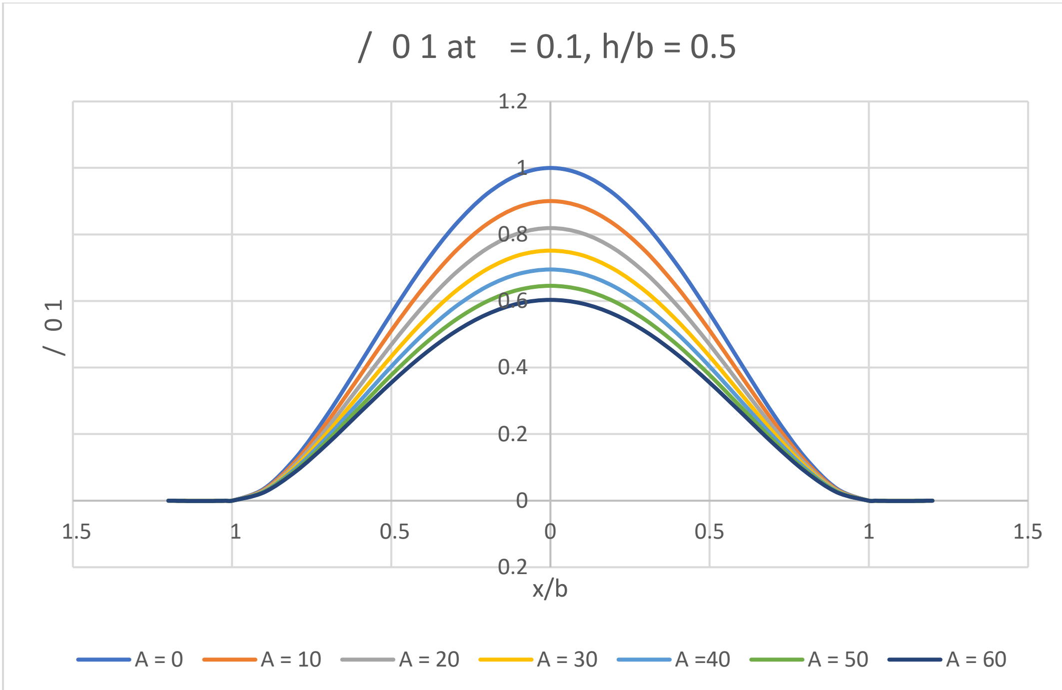


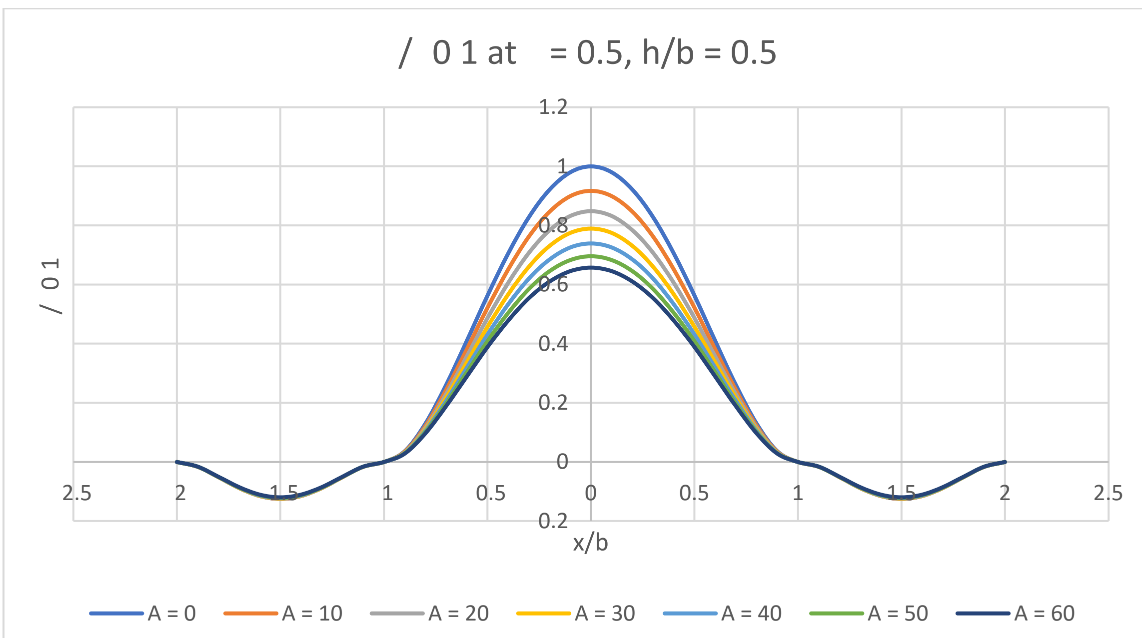
Stress 1 and reaction at = 0.5, h/b = 0.5
Stress 1
x/b
3
2.5
2
1.5
1
0.5
0
0.5
1
1.5
2
2.5
2.5
2
1.5
1
0.5
0
0.5
1
1.5
2
2.5
A = 0
A = 10
A = 20
A = 30
A =40
A = 50
A = 60



$$\frac{EI\ v(x)1}{-\frac{pb^4}{24}} = \frac{1}{\frac{\Gamma}{24}} \sum_{n=1}^{\infty} \frac{2\frac{(1+\Gamma)^2}{(\pi n)^2} + \frac{C2}{0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A} (\cos \alpha_n x - \cos \alpha_n b) \sin \alpha_n b \ldots\ldots\ldots\ldots\ldots\ldots 147$$

The deflection does not take into account the moment of inertia of the elastic media, so it is conservative. But for a media such as soils where tension is not allowed it is accurate and useful.





$$\frac{EI\,v'(x)1}{\frac{pb^3}{3}} = \frac{3}{\Gamma}\sum_{n=1}^{\infty}\frac{2(1+\Gamma)\frac{\sin\alpha_n b}{\pi n}+\frac{C2}{0.5pb^2}\frac{\pi n}{(1+\Gamma)}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3}+2k_nA}\sin\alpha_n x \ldots\ldots\ldots\ldots\ldots\ldots\ldots 148$$

Once D is found the stresses at $q = 0$ are

$$\sigma_y = -\sum_{n=\varphi}^{\infty} D\cos\alpha_n x\left[\left(a_n + b_n \frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{\alpha_n y} + \left(c_n + \frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y}\right] - p\left(1-\frac{y}{h}\right) \ldots\ldots 149$$

$$\sigma_x = k_0 \sum_{n=\varphi}^{\infty} D\cos\alpha_n x\left[\left(a_n + b_n \frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{\alpha_n y} + \left(c_n + \frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y}\right] + 2k_0 \sum_{n=\varphi}^{\infty} D\cos\alpha_n x[b_n e^{\alpha_n y} - e^{-\alpha_n y}] - k_0 p\left(1-\frac{y}{h}\right) \ldots\ldots 150$$

$$\tau_{xy} = \sum_{n=\varphi}^{\infty} D\sin\alpha_n x\left[\left((a_n+b_n) + b_n \frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{\alpha_n y} + \left(-(c_n-1) - \frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y}\right] \ldots\ldots 151$$

$$D = -p\frac{2k_n A}{\Gamma(a_n+c_n)}\frac{\frac{2(1+\Gamma)}{\pi n} + \frac{\pi n}{(1+\Gamma)}\frac{C2}{0.5pb^2}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}\sin\alpha_n b \ldots\ldots 152$$

$$a_n = -\left[c_n(1+2\alpha_n h) + 2\alpha_n^2 h^2\right]e^{-2\alpha_n h}$$

$$b_n = [2c_n - 1 + 2\alpha_n h]e^{-2\alpha_n h}$$

$$c_n = 1 + \frac{2\alpha_n^2 h^2 e^{-2\alpha_n h}}{(1-2\alpha_n h)e^{-2\alpha_n h} - 1}$$

$$d_n = a_n - b_n - c_n - 1$$

$$k_n = -\frac{a_n + c_n}{d_n}$$

Rewrite for the excel sheet

$$\sigma_y = -\sum_{n=\varphi}^{\infty} D\cos\alpha_n x\left[\left(a_{n1} + b_{n1}\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right) + \left(c_n + \frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y}\right] - p\left(1-\frac{y}{h}\right) \ldots\ldots..153$$

$$\sigma_x = k_0\sum_{n=\varphi}^{\infty} D\cos\alpha_n x\left[\left(a_{n1} + b_{n1}\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right) + \left(c_n + \frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y}\right] + 2k_0\sum_{n=\varphi}^{\infty} D\cos\alpha_n x[b_{n1} - e^{-\alpha_n y}] - k_0 p\left(1-\frac{y}{h}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots 154$$

$$\tau_{xy} = \sum_{n=\varphi}^{\infty} D\sin\alpha_n x\left[\left(a_{n1} + b_{n1}\right) + b_{n1}\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right) + \left(-(c_n - 1) - \frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots...155$$

$$a_{n1} = a_n e^{\alpha_n y} = -\left[c_n(1+2\alpha_n h) + 2\alpha_n^2 h^2\right]e^{-\alpha_n h}$$

$$b_{n1} = b_n e^{\alpha_n y} = [2c_n - 1 + 2\alpha_n h]e^{-\alpha_n h}$$

The stresses with $q = 0$ are:

Now setting the rotation to be zero at $x = \pm b$ for $p = 0$ we have:

$$0 = 3\sum_{n=1}^{\infty}\frac{\frac{W_n}{q} + \frac{2}{\Gamma}\frac{\sin\alpha_n b}{\pi n} + \frac{n\pi}{\Gamma(1+\Gamma)}\frac{C2}{0.5qb^2}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}\sin\alpha_n b$$

$$0 = \sum_{n=1}^{\infty}\frac{\frac{W_n}{q} + \frac{2}{\Gamma}\frac{\sin\alpha_n b}{\pi n}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2\beta k_n A}\sin\alpha_n b + \sum_{n=1}^{\infty}\frac{\frac{n\pi}{\Gamma(1+\Gamma)}\frac{C2}{0.5qb^2}\sin\alpha_n b\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}$$

$$\frac{C2\ 2}{0.5qb^2} = -\frac{\sum_{n=1}^{\infty}\frac{\frac{\Gamma W_n}{q}\sin\alpha_n b + 2\frac{(\sin\alpha_n b)^2}{\pi n}}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}}{\sum_{n=1}^{\infty}\frac{\frac{\pi n}{(1+\Gamma)}(\sin\alpha_n b)^2}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.156$$

Note: the maximum moment may not be at $x = 0$ depending on A

$$Rm2+ = \frac{M+}{\frac{pb^2}{6}} = 3\left[\sum_{n=1}^{\infty}\frac{1}{\Gamma}\left(\frac{\frac{2\pi n\Gamma}{(1+\Gamma)}\frac{W_n}{q}+\frac{4}{(1+\Gamma)}\sin\alpha_n b-4k_nA\frac{C2}{0.5qb^2}\frac{(1+\Gamma)}{\pi n}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3}+2k_nA}\right)+\frac{C2}{0.5qb^2}\right] \ldots\ldots 157$$

Table 30 Rm2+ verses h/b for   = 0.1

h/b	A = 0	A = 10	A = 20	A = 30	A = 40	A = 50	A = 60
1E 13	0.999951	0.999951	0.999951	0.999951	0.99995096	0.999951	0.999951
0.1	1.000115	0.998547	0.996984	0.995426	0.99387263	0.992325	0.9907813
0.2	1.000922	0.989317	0.977982	0.966907	0.95608366	0.945504	0.935159
0.3	1.002199	0.966875	0.933934	0.903146	0.87430978	0.847248	0.8218049
0.4	1.002979	0.930966	0.868214	0.813089	0.76431422	0.72088	0.6819768
0.5	1.001226	0.885941	0.793031	0.716717	0.65303264	0.599169	0.5530826
0.6	0.995996	0.838496	0.720961	0.630219	0.55827295	0.499993	0.4519402
0.7	0.987893	0.794486	0.659579	0.560616	0.48525627	0.426187	0.3788059
0.8	0.978185	0.757132	0.611111	0.50811	0.43198114	0.37371	0.3278726
0.9	0.968082	0.727237	0.574573	0.469914	0.39418369	0.337171	0.2929271
1	0.958444	0.704185	0.547764	0.442664	0.36772175	0.311941	0.2690511
1.5	0.926194	0.651872	0.492221	0.388759	0.31687964	0.264441	0.2247715
2	0.914313	0.639852	0.481194	0.378821	0.30792247	0.256327	0.217375

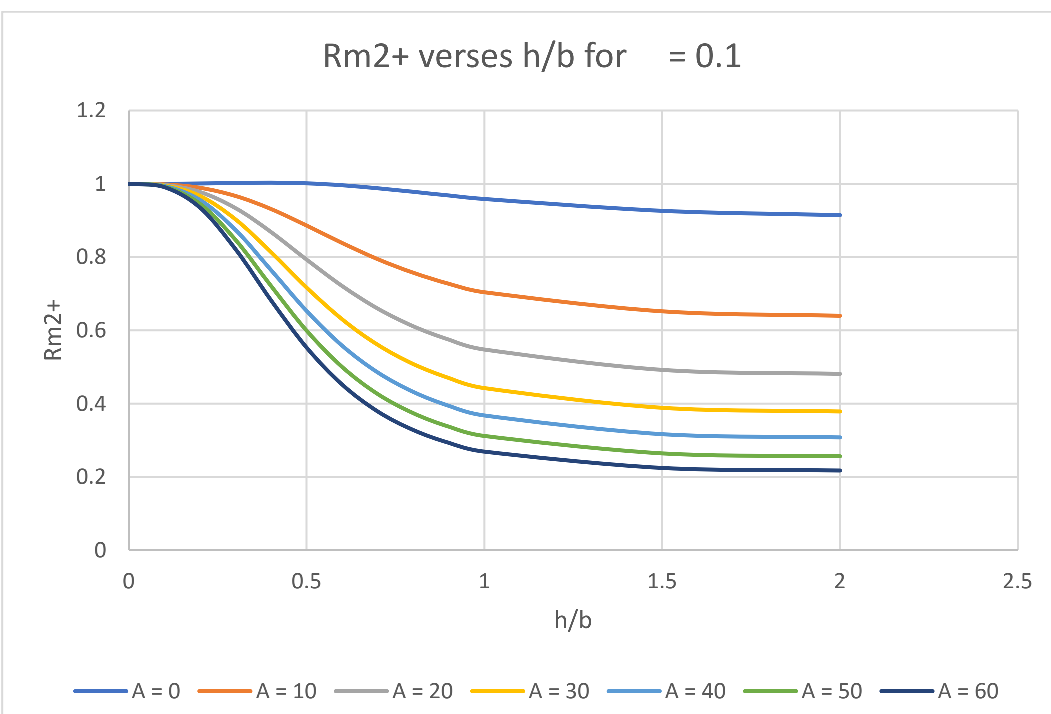
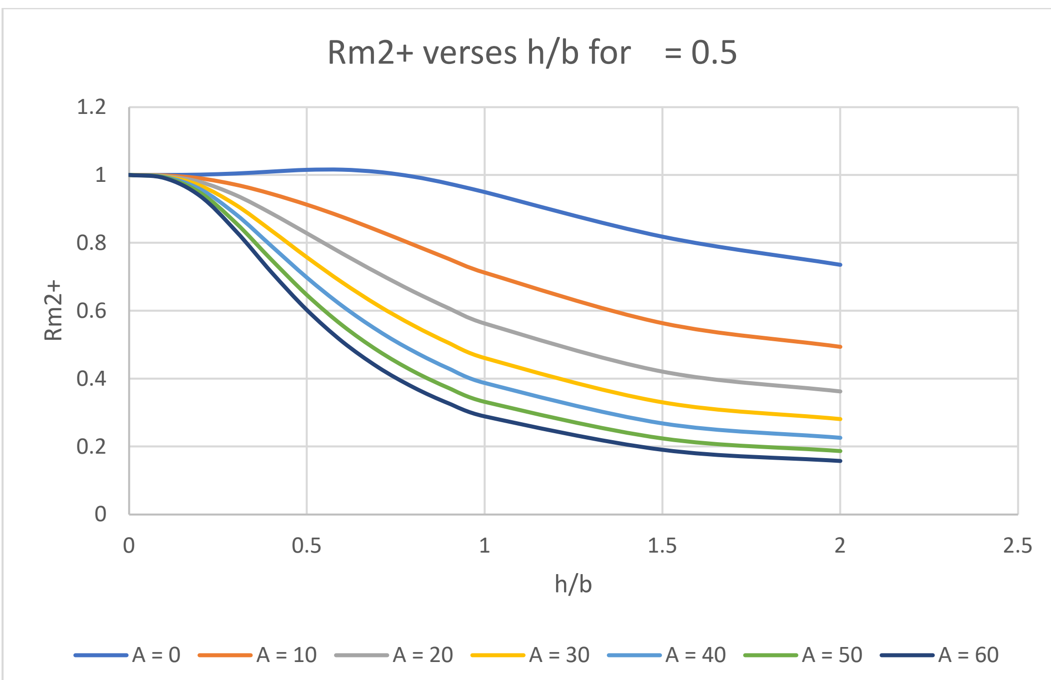


Table 31 Rm2+ verses h/b for = 0.5

h/b	A = 0	A = 10	A = 20	A = 30	A = 40	A = 50	A = 60
1E 13	0.999909	0.999909	0.999909	0.999909	0.99990882	0.999909	0.9999088
0.1	1.000075	0.998537	0.997004	0.995477	0.99395379	0.992436	0.9909225
0.2	1.001236	0.990094	0.979211	0.968579	0.95818814	0.948031	0.9380988
0.3	1.00447	0.971484	0.940678	0.911845	0.88480226	0.859389	0.8354625
0.4	1.010034	0.944653	0.887224	0.836401	0.79112512	0.75055	0.7139909
0.5	1.015145	0.912496	0.828087	0.757544	0.69777974	0.646551	0.6021921
0.6	1.015705	0.876153	0.768512	0.683175	0.61401449	0.55694	0.5091207
0.7	1.009098	0.836346	0.710805	0.61582	0.54169997	0.482428	0.4340767
0.8	0.994937	0.794493	0.656503	0.556245	0.48045463	0.421386	0.374221
0.9	0.974435	0.75251	0.606878	0.50464	0.42934376	0.371864	0.3267393
1	0.949544	0.712205	0.562738	0.460747	0.38719796	0.331966	0.28918
1.5	0.818343	0.563711	0.420807	0.330314	0.2684521	0.223856	0.1904226
2	0.735459	0.493921	0.362498	0.280867	0.22582275	0.186559	0.1573764



$$Rm2 - = \frac{M -}{\frac{-pb^2}{3}} = -\frac{3}{2}\left[\sum_{n=1}^{\infty}\frac{1}{\Gamma}\left(\frac{\frac{2\pi n\Gamma}{(1+\Gamma)}\frac{W_n}{q}+\frac{4}{(1+\Gamma)}\sin\alpha_n b-4k_nA\frac{C2}{0.5qb^2}\frac{(1+\Gamma)}{\pi n}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3}+2k_nA}\cos\alpha_n b\right)+\frac{C2}{0.5qb^2}\right] \quad \ldots\ldots 158$$

Table 32 Rm2 verses h/b for = 0.1

h/b	A = 0	A = 10	A = 20	A = 30	A = 40	A = 50	A = 60
1E 13	1.000025	1.000025	1.000025	1.000025	1.00002452	1.000025	1.0000245
0.1	0.999892	0.99915	0.998411	0.997673	0.99693662	0.996202	0.9954697
0.2	0.997645	0.991677	0.985812	0.980047	0.97437772	0.968803	0.9633198
0.3	0.99255	0.973146	0.954763	0.937318	0.92073191	0.904939	0.8898776
0.4	0.985821	0.945045	0.90863	0.875865	0.84619108	0.819159	0.7944068
0.5	0.978196	0.91188	0.856781	0.810136	0.77003524	0.735112	0.7043624
0.6	0.970059	0.87862	0.808015	0.751585	0.7052618	0.666417	0.6332733
0.7	0.961768	0.848841	0.767144	0.704899	0.65562992	0.615473	0.581979
0.8	0.953724	0.824182	0.735283	0.669987	0.61966203	0.579461	0.5464459
0.9	0.94628	0.804796	0.711509	0.644787	0.59431423	0.554543	0.5222171
1	0.939663	0.790049	0.694214	0.626931	0.57667906	0.537442	0.505764
1.5	0.919306	0.757311	0.658924	0.592072	0.54321474	0.505638	0.4756272
2	0.912214	0.749992	0.652074	0.585781	0.53744187	0.500319	0.4707014

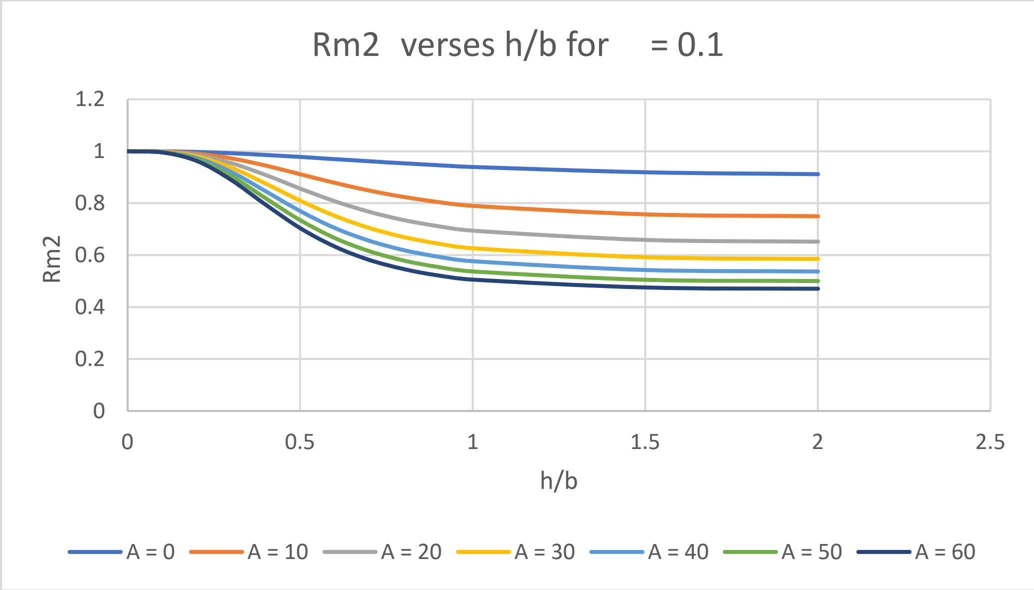
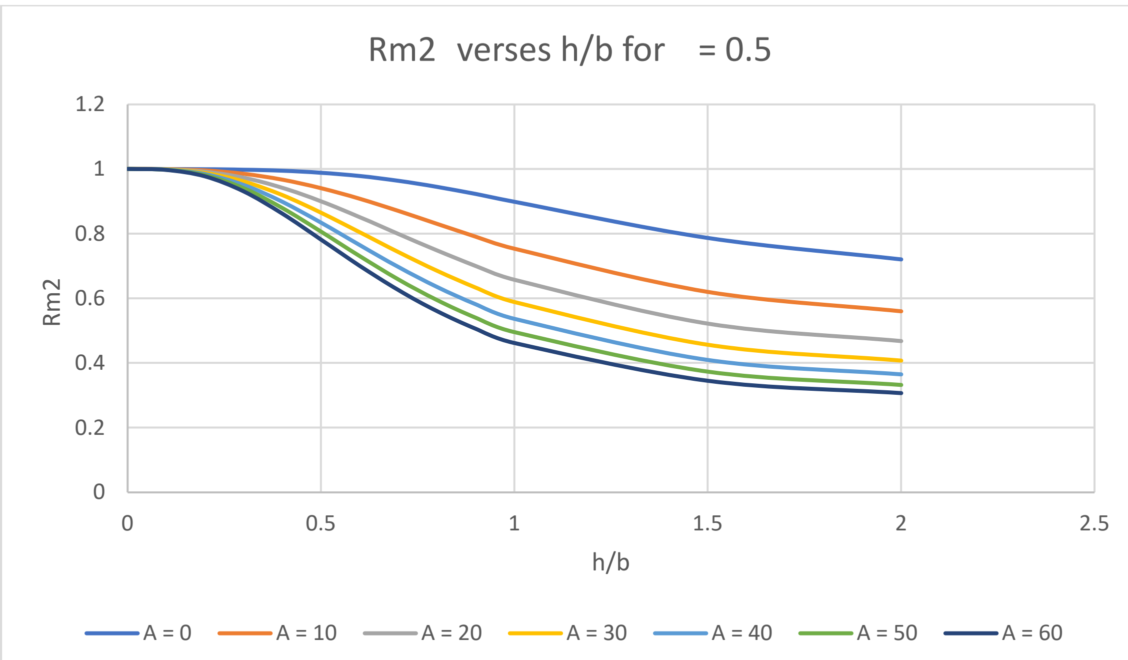


Table 33 Rm2 verses h/b for   = 0.5

h/b	A = 0	A = 10	A = 20	A = 30	A = 40	A = 50	A = 60
1E 13	1.000046	1.000046	1.000046	1.000046	1.00004559	1.000046	1.0000456
0.1	0.999963	0.999498	0.999035	0.998573	0.99811294	0.997654	0.997196
0.2	0.999383	0.995526	0.991745	0.988037	0.9844002	0.980832	0.9773291
0.3	0.997824	0.985128	0.973116	0.961726	0.95090338	0.940601	0.9307767
0.4	0.994609	0.966916	0.941959	0.919302	0.89860103	0.879578	0.862007
0.5	0.988517	0.940873	0.900237	0.86502	0.83408927	0.806617	0.7819828
0.6	0.978367	0.908028	0.851372	0.804467	0.76478381	0.73062	0.7007833
0.7	0.963673	0.870403	0.799395	0.743092	0.69705725	0.658509	0.6256089
0.8	0.944797	0.830558	0.748089	0.685201	0.63529749	0.59449	0.56033
0.9	0.922733	0.790946	0.700263	0.633409	0.5816771	0.540194	0.5060076
1	0.898769	0.753447	0.657515	0.588755	0.53662586	0.495469	0.4619664
1.5	0.78691	0.620045	0.521875	0.456457	0.40929672	0.373411	0.34501
2	0.720646	0.559773	0.467866	0.407656	0.36472359	0.3323	0.3067786



Rv2, the stress and deflections are the same just C2 is different

$$Rv2=\frac{\int\sigma_y}{-qb}$$
$$=\sum_{n=1}^{\infty}\left(\frac{+\frac{W_n}{q}\frac{(\pi n)^2}{(1+\Gamma)^2}+\frac{2}{\Gamma}\frac{\pi n}{(1+\Gamma)^2}\sin\alpha_n b-2k_nA\frac{1}{\Gamma}\frac{C2}{0.5qb^2}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3}+2k_nA}\right)\sin\alpha_n b^* \dots\dots\dots\dots.159$$

And the stress at $x = 0$ we have

Table 34 Rv 2 verses h/b for   = 0.1, b = b*

h/b	A = 0	A = 10	A = 20	A = 30	A = 40	A = 50	A = 60
1E 13	0.999755	0.999755	0.999755	0.999755	0.99975487	0.999755	0.9997549
0.1	0.985215	0.988204	0.991185	0.994158	0.99712304	1.00008	1.0030292
0.2	0.966849	0.973095	0.979205	0.985183	0.99103492	0.996763	1.002371
0.3	0.951827	0.956682	0.961149	0.965266	0.96906474	0.972574	0.9758186
0.4	0.941455	0.941291	0.940826	0.940124	0.93923742	0.938205	0.9370565
0.5	0.934249	0.927336	0.921055	0.915285	0.90993902	0.904952	0.9002726
0.6	0.92902	0.91524	0.903871	0.894194	0.88576276	0.878285	0.8715588
0.7	0.925027	0.905283	0.890127	0.87789	0.86764869	0.858845	0.8511223
0.8	0.92186	0.897451	0.879727	0.865955	0.85474153	0.845298	0.8371429
0.9	0.919299	0.891496	0.872124	0.857473	0.84576337	0.836033	0.8277131
1	0.917218	0.88707	0.866675	0.851533	0.83958133	0.829735	0.8213672
1.5	0.911498	0.87757	0.855806	0.840153	0.82804505	0.818203	0.8099158
2	0.909658	0.875525	0.853757	0.838151	0.82610163	0.816318	0.8080845

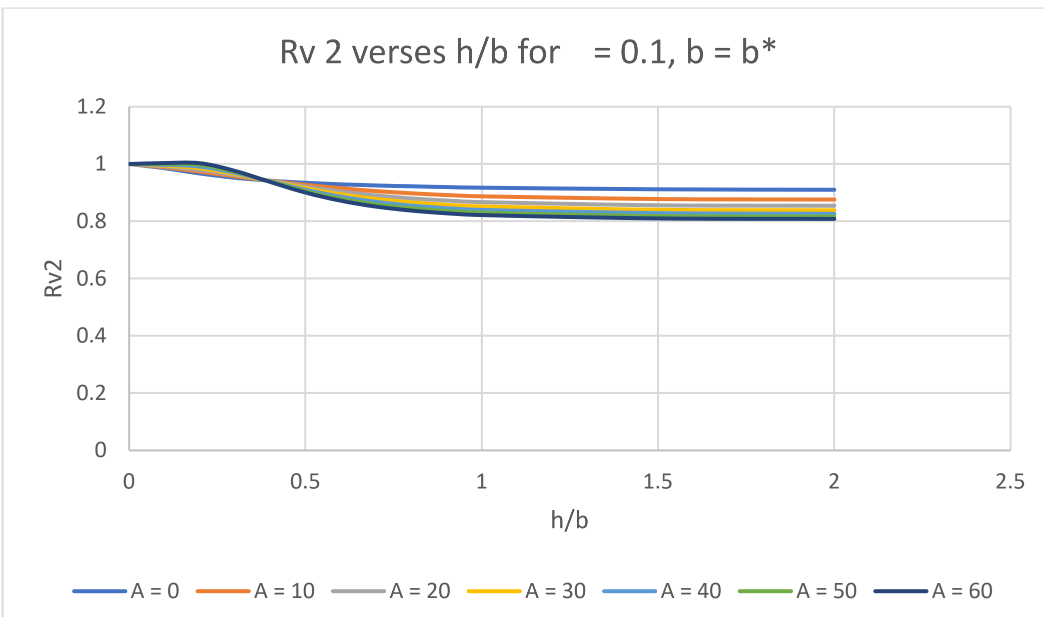
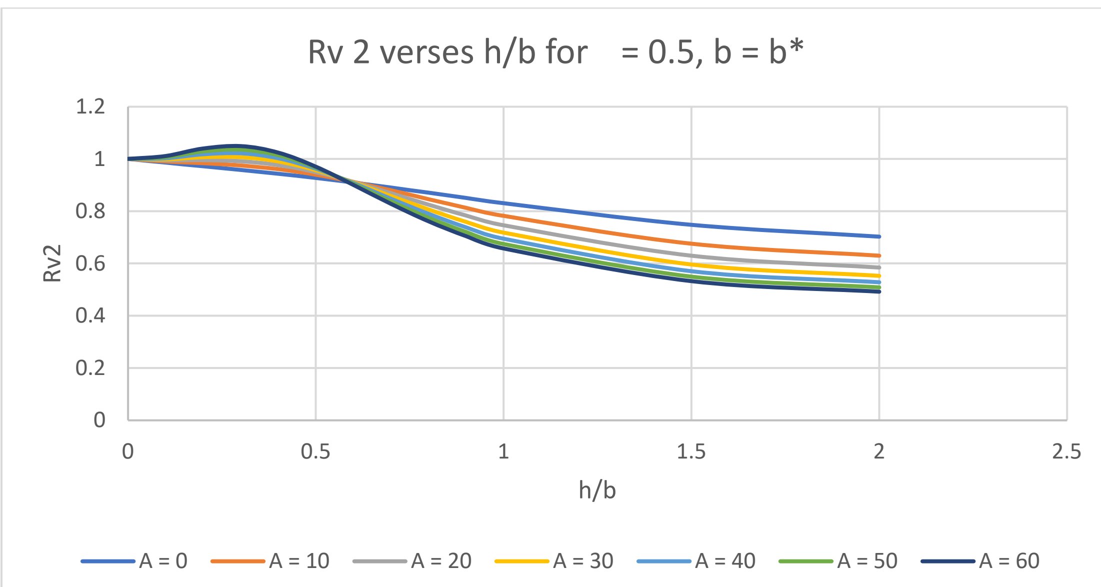


Table 35 Rv 2 verses h/b for = 0.5, b = b*

h/b	A = 0	A = 10	A = 20	A = 30	A = 40	A = 50	A = 60
1E 13	0.999909	0.999909	0.999909	0.999909	0.99990883	0.999909	0.9999088
0.1	0.985886	0.990138	0.994382	0.998618	1.00284491	1.007063	1.0112731
0.2	0.971833	0.98364	0.995266	1.006716	1.01799517	1.029108	1.0400582
0.3	0.957787	0.974905	0.991179	1.006678	1.02146154	1.035583	1.0490895
0.4	0.943192	0.960844	0.976478	0.990414	1.00290753	1.014163	1.0243483
0.5	0.927158	0.939756	0.949414	0.95686	0.96260191	0.967012	0.9703628
0.6	0.90952	0.912056	0.912083	0.91048	0.90779517	0.904384	0.9004858
0.7	0.890587	0.879939	0.869068	0.85831	0.84782816	0.837701	0.8279602
0.8	0.870787	0.846125	0.825072	0.806581	0.79003625	0.775042	0.7613247
0.9	0.850617	0.812949	0.783515	0.759239	0.73851278	0.720397	0.704293
1	0.830625	0.782014	0.746289	0.718062	0.69471377	0.674797	0.6574295
1.5	0.747966	0.675728	0.629668	0.596503	0.57081036	0.549934	0.5324004
2	0.702596	0.629614	0.584516	0.552628	0.52821315	0.508533	0.4921012



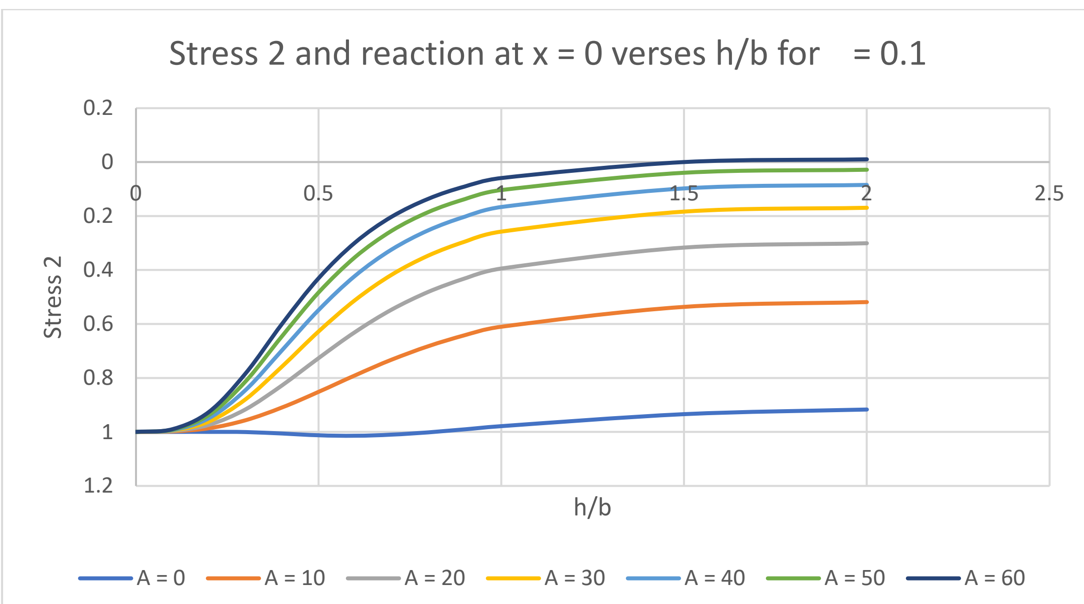
$$\frac{\sigma_y}{q} = \frac{sft2}{q} = \sum_{n=1}^{\infty} 2k_n A \left(\frac{+\frac{W_n}{q} + \frac{2}{\Gamma}\frac{\sin\alpha_n b}{n\pi} + \frac{\pi n}{\Gamma(1+\Gamma)}\frac{C2}{0.5qb^2}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A} \right) - \sum_{n=1}^{\infty}\left(\frac{W_n}{q} + \frac{2}{\Gamma}\frac{\sin\alpha_n b}{n\pi}\right) \dots\dots 160$$

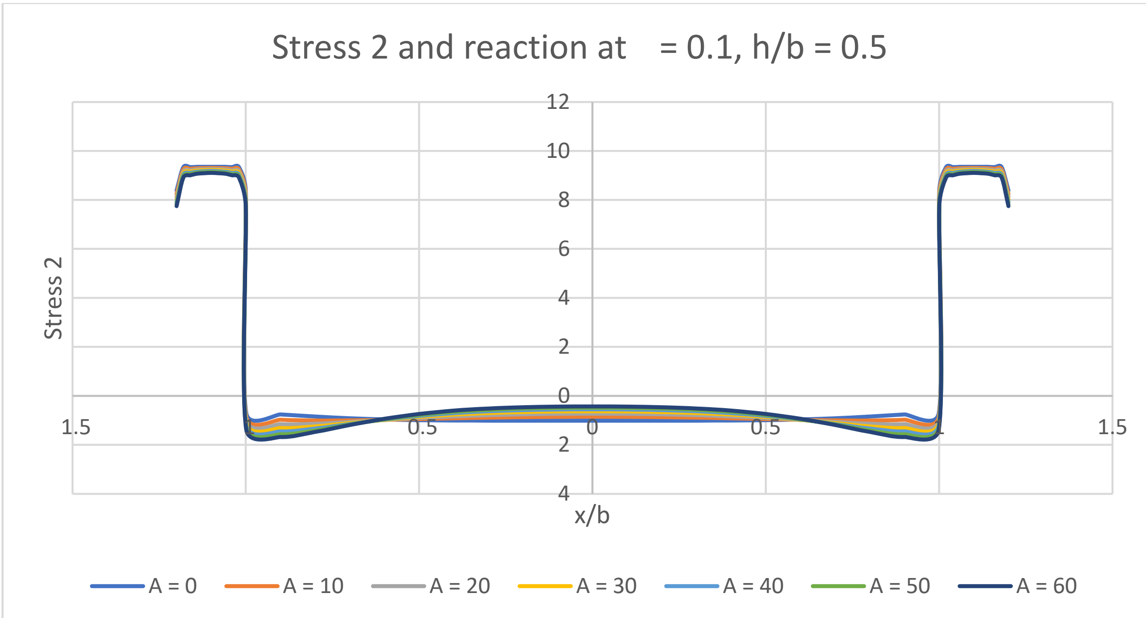
And the stress at $x = b + t$ we have

$$Reaction = \sum_{n=1}^{\infty} 2k_n A \left(\frac{\frac{W_n}{q} + \frac{2}{\Gamma}\frac{\sin \alpha_n b}{n\pi} + \frac{\pi n}{\Gamma(1+\Gamma)}\frac{C2}{0.5qb^2}\sin \alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A} \right) \cos \alpha_n n\pi - \sum_{n=1}^{\infty} \left(\frac{W_n}{q} + \frac{2}{\Gamma}\frac{\sin \alpha_n b}{n\pi} \right) \cos \alpha_n n\pi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 161$$

Table 36 Stress 2 at x = 0 verses h/b for = 0.1

h/b	A = 0	A = 10	A = 20	A = 30	A = 40	A = 50	A = 60
1E 13	0.99932	0.99932	0.99932	0.99932	0.9993212	0.99932	0.999321
0.1	0.99938	0.99772	0.99606	0.99441	0.9927604	0.99112	0.989482
0.2	0.99934	0.98617	0.97334	0.96085	0.9486707	0.9368	0.925223
0.3	1.00008	0.95615	0.91579	0.87861	0.844263	0.81246	0.782946
0.4	1.00565	0.90896	0.82704	0.75706	0.6968191	0.6446	0.599061
0.5	1.01221	0.85111	0.72597	0.62697	0.5474656	0.48277	0.429533
0.6	1.01401	0.78973	0.62924	0.51071	0.4210125	0.35179	0.29751
0.7	1.00978	0.73185	0.54656	0.41718	0.3237566	0.25458	0.202333
0.8	1.00104	0.68192	0.48089	0.34642	0.2526878	0.18538	0.135972
0.9	0.98992	0.64139	0.43106	0.29473	0.2020917	0.13704	0.090283
1	0.97814	0.60975	0.39425	0.25766	0.1665082	0.10351	0.058921
1.5	0.93391	0.53617	0.31676	0.18334	0.097263	0.03959	4.009E 06
2	0.91639	0.51867	0.30092	0.16926	0.084728	0.02836	0.010134




Stress 2 and reaction at = 0.1, h/b = 0.5
Stress 2
x/b
12
10
8
6
4
2
0
2
4
1.5
0.5
0
0.5
1.5
A = 0
A = 10
A = 20
A = 30
A = 40
A = 50
A = 60

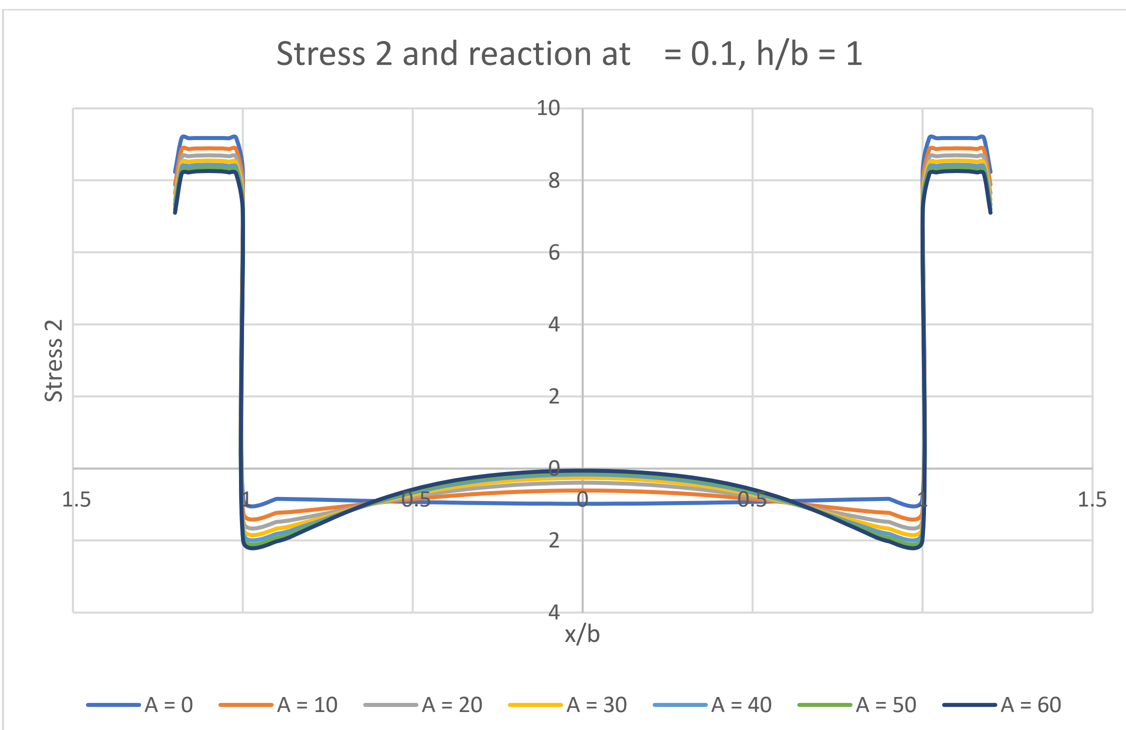
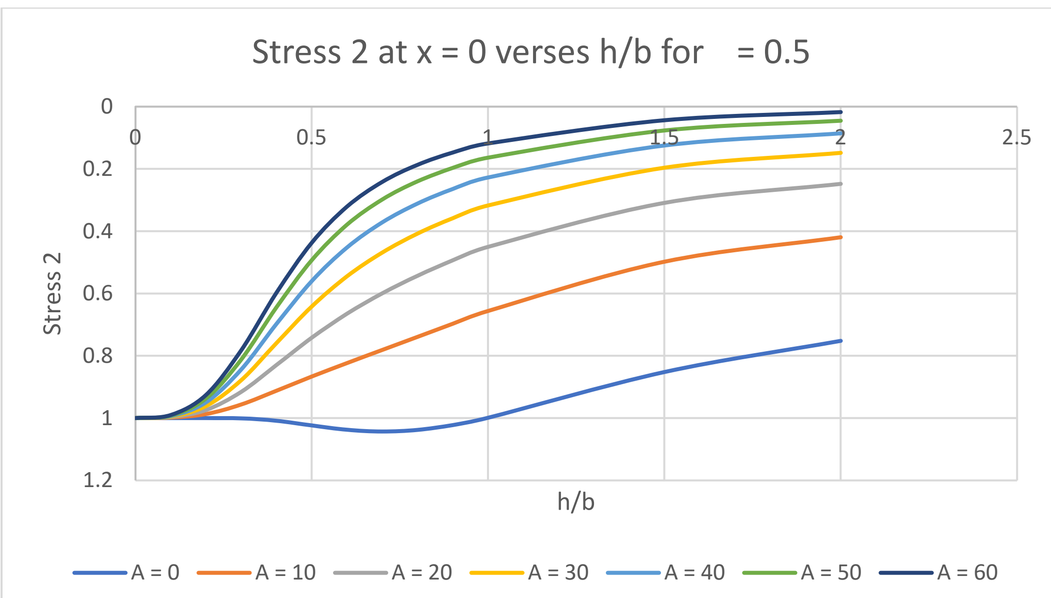
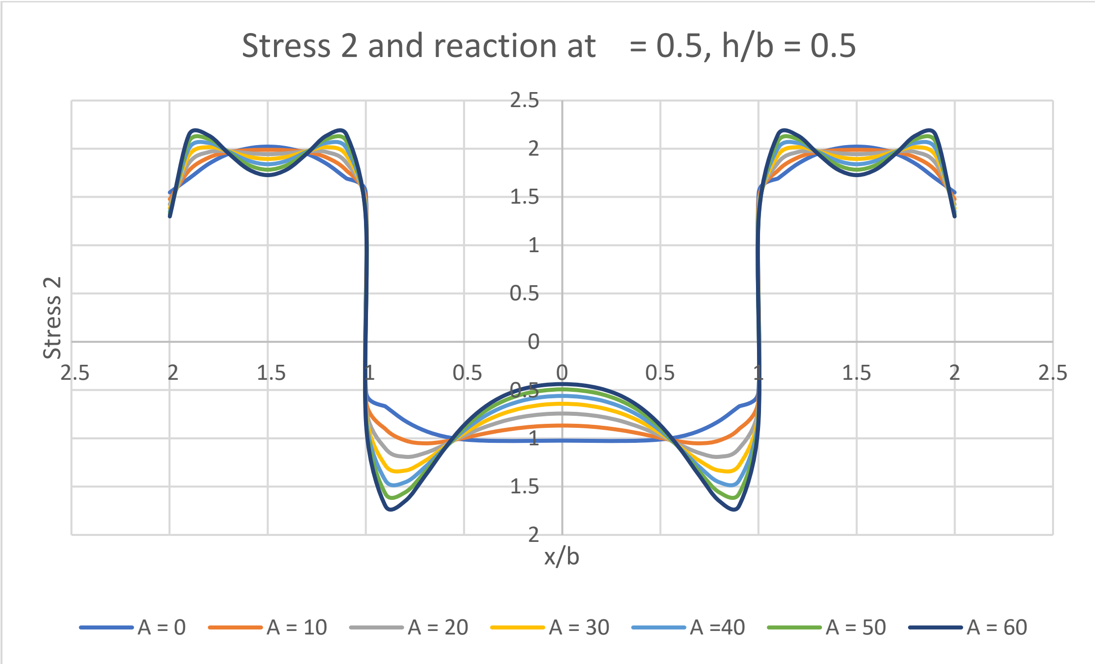
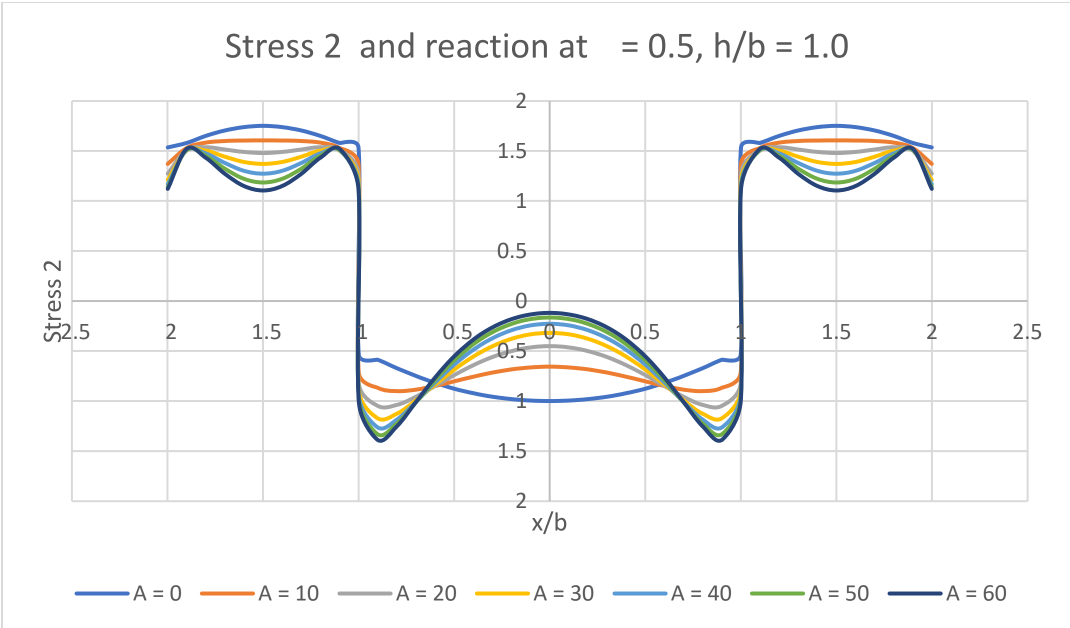

Stress 2 and reaction at = 0.1, h/b = 1
Stress 2
x/b
10
8
6
4
2
0
2
4
1.5
0.5
0
0.5
1
1.5
A = 0
A = 10
A = 20
A = 30
A = 40
A = 50
A = 60

Table 37 Stress 2 at x = 0 verses h/b for   = 0.5

h/b	A = 0	A = 10	A = 20	A = 30	A = 40	A = 50	A = 60
1E 13	0.99989	0.99989	0.99989	0.99989	0.9998898	0.99989	0.99989
0.1	0.99993	0.99826	0.9966	0.99495	0.9933039	0.99166	0.990025
0.2	0.99988	0.98671	0.97388	0.96138	0.9491991	0.93733	0.925749
0.3	1.00049	0.95634	0.91574	0.87831	0.8437173	0.81168	0.781942
0.4	1.00831	0.9119	0.8296	0.75884	0.6975885	0.64426	0.597577
0.5	1.0234	0.86624	0.74171	0.64156	0.5599918	0.49283	0.437022
0.6	1.03734	0.82317	0.66485	0.54483	0.4520182	0.37907	0.320973
0.7	1.04297	0.7812	0.59945	0.46853	0.3715531	0.29815	0.241633
0.8	1.03763	0.73901	0.54297	0.40772	0.3110309	0.24003	0.186802
0.9	1.02208	0.69685	0.49353	0.35826	0.2642978	0.19695	0.14753
1	0.99879	0.65584	0.45024	0.31747	0.2273597	0.16402	0.118324
1.5	0.85212	0.4977	0.30891	0.19644	0.12471	0.07687	0.043986
2	0.75185	0.41933	0.24809	0.14846	0.0861497	0.04535	0.017833

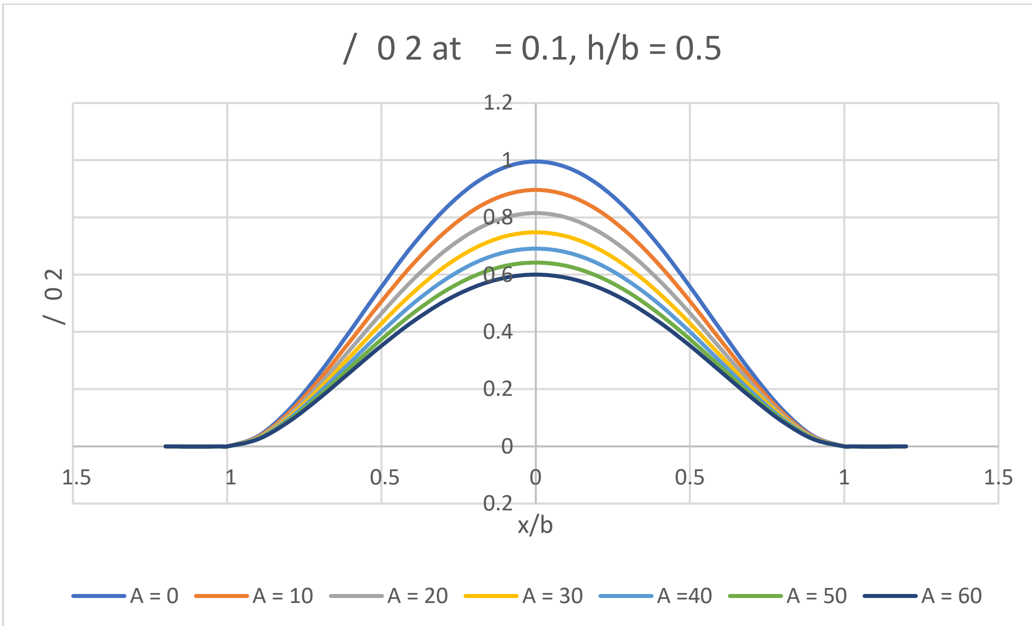


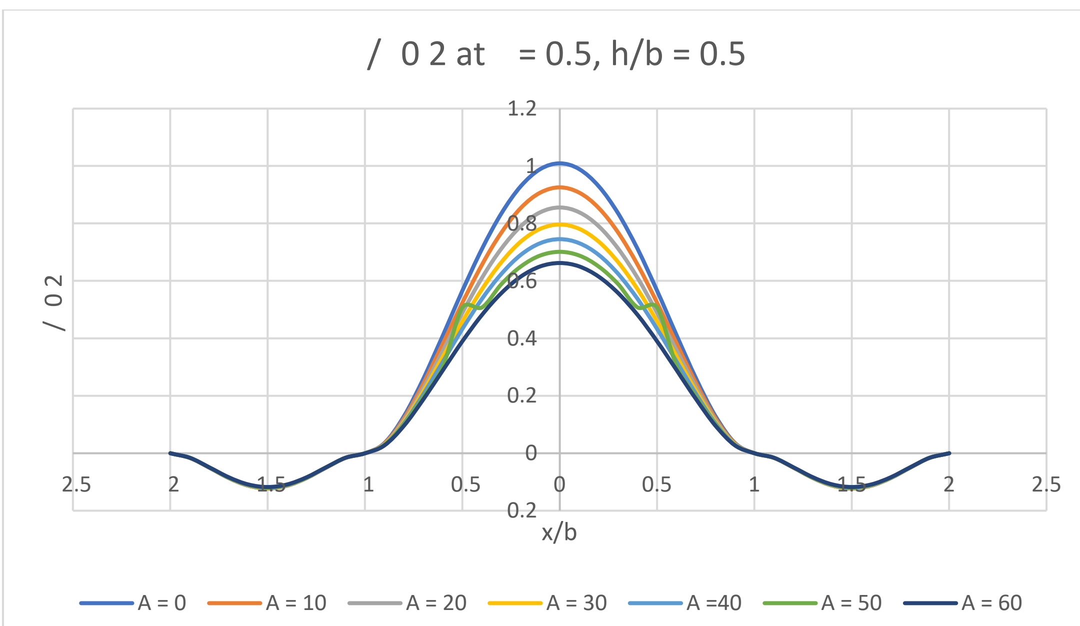

Stress 2 and reaction at = 0.5, h/b = 0.5
Stress 2
x/b
A = 0
A = 10
A = 20
A = 30
A =40
A = 50
A = 60


Stress 2 and reaction at = 0.5, h/b = 1.0
Stress 2
x/b
A = 0
A = 10
A = 20
A = 30
A = 40
A = 50
A = 60

$$\frac{EI\ v(x)2}{-\frac{qb^4}{24}} = 24\sum_{n=1}^{\infty}\frac{\frac{W_n}{q}\frac{(1+\Gamma)}{\pi n} + 2\frac{(1+\Gamma)}{\Gamma}\frac{\sin\alpha_n b}{(\pi n)^2} + \frac{1}{\Gamma}\frac{C2}{0.5qb^2}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}(\cos\alpha_n x - \cos\alpha_n b) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..162$$

The deflection does not take into account the moment of inertia of the elastic media, so it is conservative. But for a media such as soils where tension is not allowed it is accurate and useful.





$$\frac{EI\,v'(x)2}{\frac{qb^3}{3}} = 3\sum_{n=1}^{\infty} \frac{\frac{W_n}{q} + \frac{2}{\Gamma}\frac{\sin\alpha_n b}{\pi n} + \frac{n\pi}{\Gamma(1+\Gamma)}\frac{C2}{0.5qb^2}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}\sin\alpha_n x \ldots\ldots\ldots\ldots\ldots\ldots..163$$

Once D is found the stresses with $q = 0$ are:

$$\sigma_y = -\sum_{n=\varphi}^{\infty} cos\,\alpha_n x\left[\left(a_n D + w_{n1}w_n + (b_n D + w_{n2}w_n)\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{\alpha_n y} + \left(c_n D + w_{n3}w_n + D\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y} + \frac{2q}{\Gamma}\frac{\sin\alpha_n b}{n\pi}\right] \quad -b < x < b \ldots\ldots 164$$

$$\sigma_x = k_0\sum_{n=\varphi}^{\infty} cos\,\alpha_n x\left[\left(a_n D + w_{n1}w_n + (b_n D + w_{n2}w_n)\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{\alpha_n y} + \left(c_n D + w_{n3}w_n + D\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y} - \frac{2q}{\Gamma}\frac{\sin\alpha_n b}{n\pi}\right] + 2k_0\sum_{n=\varphi}^{\infty} cos\,\alpha_n x[(b_n D + w_{n2}w_n)e^{\alpha_n y} - De^{-\alpha_n y}] \quad -b < x < b \ldots\ldots.165$$

$$\tau_{xy} = \sum_{n=\varphi}^{\infty} sin\,\alpha_n x\left[\left((a_n + b_n)D + w_{n1}w_n + w_{n2}w_n + (b_n D + w_{n2}w_n)\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{\alpha_n y} + \left(-(c_n - 1)D - w_{n3}w_n - D\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y}\right] \quad -b < x < b \ldots\ldots\ldots 166$$

$$D = -\frac{s\alpha_n v_n}{\beta d_n} - \frac{W1_n}{d_n} = -q\frac{2k_n A}{(a_n + c_n)}\left(\frac{+\frac{W_n}{q} + \frac{2}{\Gamma}\frac{\sin\alpha_n b}{n\pi} + \frac{\pi n}{\Gamma(1+\Gamma)}\frac{C2}{0.5qb^2}\sin\alpha_n b}{\frac{(\pi n)^3}{(1+\Gamma)^3} + 2k_n A}\right) - \frac{W1_n}{d_n} \ldots 167$$

Where:

$W1_n = (w_{n1} - w_{n2} - w_{n3})w_n$

$a_n = -\left[c_n(1 + 2\alpha_n h) + 2\alpha_n^2 h^2\right]e^{-2\alpha_n h}$

$b_n = [2c_n - 1 + 2\alpha_n h]e^{-2\alpha_n h}$

$$c_n = 1 + \frac{2\alpha_n^2 h^2 e^{-2\alpha_n h}}{(1-2\alpha_n h)e^{-2\alpha_n h} - 1}$$

$$d_n = a_n - b_n - c_n - 1$$

$$k_n = -\frac{a_n + c_n}{d_n}$$

$$W_n = -\frac{(a_n + c_n)(w_{n1} - w_{n2} - w_{n3})w_n}{(a_n - b_n - c_n - 1)} + (w_{n1} + w_{n3})w_n$$

$$w_{n1} = -w_{n3}(1 + 2\alpha_n h)e^{-2\alpha_n h} - (1 + \alpha_n h)e^{-\alpha_n h}$$

$$w_{n2} = 2w_{n3}e^{-2\alpha_n h} + e^{-\alpha_n h}$$

$$w_{n3} = \frac{\alpha_n h e^{-\alpha_n h}}{(1-2\alpha_n h)e^{-2\alpha_n h} - 1}$$

Rewrite for the excel sheet

$$\sigma_y = -\sum_{n=\varphi}^{\infty} cos\,\alpha_n\, x\left[\left(a_{n1}D + w_{n11}w_n + (b_{n1}D + w_{n21}w_n)\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right) + \left(c_n D + w_{n3}w_n + D\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y} + \frac{2q}{\Gamma}\frac{\sin\alpha_n b}{n\pi}\right] \quad -b < x < b \ldots\ldots.168$$

$$\sigma_x = k_0\sum_{n=\varphi}^{\infty} cos\,\alpha_n\, x\left[\left(a_{n1}D + w_{n11}w_n + (b_{n1}D + w_{n21}w_n)\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right) + \left(c_n D + w_{n3}w_n + D\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y} - \frac{2q}{\Gamma}\frac{\sin\alpha_n b}{n\pi}\right] + 2k_0\sum_{n=\varphi}^{\infty} cos\,\alpha_n\, x[(b_{n1}D + w_{n21}w_n) - De^{-\alpha_n y}] \quad -b < x < b \ldots\ldots.169$$

$$\tau_{xy} = \sum_{n=\varphi}^{\infty} sin\,\alpha_n\, x\left[\left((a_{n1} + b_{n1})D + w_{n11}w_n + w_{n21}w_n + (b_{n1}D + w_{n21}w_n)\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{\alpha_n y} + \left(-(c_n - 1)D - w_{n3}w_n - D\frac{\pi n}{(1+\Gamma)}\frac{y}{b}\right)e^{-\alpha_n y}\right] \quad -b < x < b \ldots\ldots\ldots.170$$

$$a_{n1} = a_n e^{\alpha_n h} = -[c_n(1 + 2\alpha_n h) + 2\alpha_n^2 h^2]e^{-\alpha_n h}$$

$$b_{n1} = b_n e^{\alpha_n h} = [2c_n - 1 + 2\alpha_n h]e^{-\alpha_n h}$$

$$w_{n11} = w_{n1}e^{\alpha_n h} = -w_{n3}(1 + 2\alpha_n h)e^{-\alpha_n h} - (1 + \alpha_n h)$$

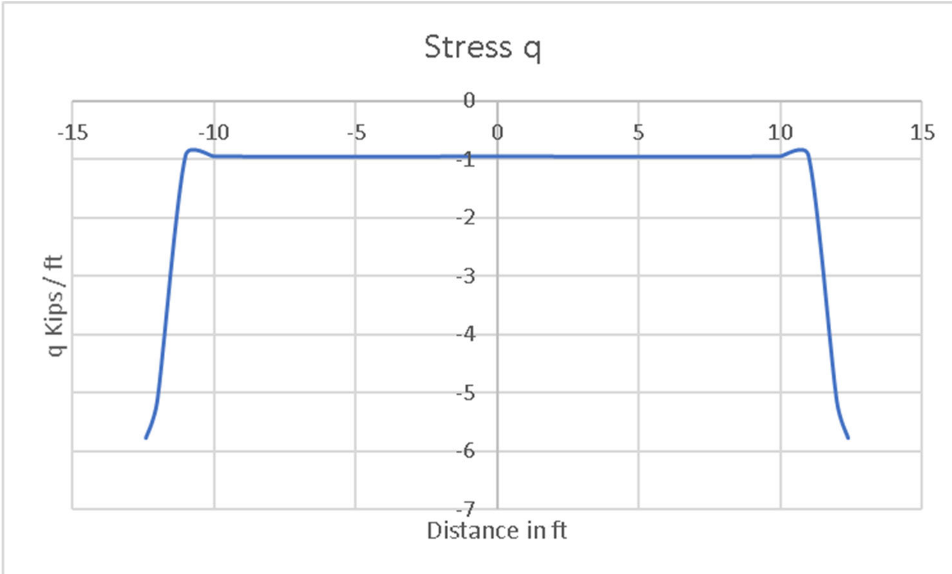
$$w_{n21} = w_{n2}e^{\alpha_n h} = 2w_{n3}e^{-\alpha_n h} + 1$$

Example 6:
In example 2 we used a 30 ft wall to approximate an infinite wall. When redoing the calculation with h/b = 15 and A = 141 *Rm-* = 0.2846 is close to the example = 0.28453 but *Rv* = 0.59241 verses 0.592369. For A = 20 *Rm-* = 0.6484456 is close to the example *Rm-* = 0.648373 but *Rv* = 0.833056 verses 0.833077. Part of the reason is in example 1, *n* max was 20,000 where in the posted excel sheet *n* max was 5,000 and *h* has an effect.

Example 7:
A concrete culvert has a top deck slab 15 ft span fixed end 12” thick with 10 ft of soils on top. The load is 100 psf live load and dead load of soils + beam = 120x10 +150 = 1350 psf we use s = 12” =0.3 $1/\beta = E_s/(1-\nu^2)$ =1111 psi Ec = E = 57000 3000 = 3,122,018.58 psi. So, A = 0.900769, h/b = 1.3333 and t/b = 0.5. Rm 1 = 0.9770204 Rm 2 = 0.8005702. So factored negative moment = 1.6*(1.350*0.9770204 +0.100*0.8005702) *15*15/12 = 41.97 Kips ft. Rm 1 + = 0.9639418 Rm 2 + = 0.8281587. The factored positive moment = 1.6*(1.350*0.9639418+0.100*0.8281587) *15*15/24 = 20.76 Kips ft. The shear at $x = 0.867b$ Rv 1 = 0.8526281 Rv 2 = 0.6817198. The factored shear = 1.6*(1.350*0.8526281 + 0.100*0.6817198) *15/2 = 14.63 kips and the top deck ready to be designed provide adding the axial load. We note that all the stresses are in compression in the region $b < x < b$ and $0 < y < 9.75$ ft. And because we assumed t/b = 0.5 for x>b the deflection is zero then for x>b, all stresses are in compression and for b<x<b is all compression then this solution is applicable for soil.

Example 8:

An existing floor sitting on a steel beam W14x30. The span is 25 ft q = 1.6 kip/ft was designed with ASD the old methods. It is a fixed end beam. Thus, Mmax = $1.6*25^2/12$ = 83.33 kip ft. The required section modulus for a compact beam is = 83.33*12/24 = 41.67 < 42 in^3 The properties of the beam A = 8.85 in^2 Sx = 42 in^3 and I = 291 in^4. The beam sits on HSS6x6 column.
In a renovation the floor was elevated 6 inches by the architect request so a 6-inch plate one inch thick was used to tack weld to the steel beam usually it is a T section. We will demonstrate the load is not uniform anymore and arching reduction takes place. We have A = 5799 t/b = 0.02 b = 12.5 ν = 0.3. Es = E = 29,000,000 psi and they cancel each other in A. We find that *Rm*2- = 0.9679295 a 3.2% reduction. See graph for the new distribution of stress, so it is not uniform anymore.



Example 9:
A glulam beam is fixed at the ends. The beam is 5.5" x 24" spans 40 ft to support a floor load q = 468 lbs/ft and the weight of the beam p = 32.08 lbs/ft. Thus, the moment is $(q + p)$ $L^2/12$ = 66,677.78 ft-lbs. The required section modulus for Fb = 2400 psi is 333.39 in^3 Fv = 165 psi and the required area = 81.83 in^2 the shear a 2 ft from the support or 0.9b is 9001.44 lbs this is with no arching. The section modulus of the beam is 528 in^3 and the area = 132 in^2, the beam was chosen to give 0.51-inch deflection at the center, so the beam is OK using conventional methods. Now we consider arching so we take 17 inches for h the mass media and 7 inch is the beam under the mass media. Giving I = 157.21 in^4 and E cancels Es so A = 241819.2 with s = 5.5-inch b = 240 inches. We assume the column is a HSS3x3 with a 4"x 4" plate for a 20003.2 lbs reaction. Thus $\Gamma = t/b$ = 2/240 = 0.008333 and h/b = 17/240 = 0.0708. This gives $Rm1-$ = 0.1232561 and $Rv1$ = 0.2922825 taken at x = 0.9708b $Rm2-$ = 0.1230694 and $Rv2$ = 0.2919907. The section modulus for a 7-inch beam is 44.92 in^3 and the area = 38.5 in^2. Thus, the moment = (32.03*0.1232561+ 468*0.1230694) *40^2 / 12 = 8205.91 lbs ft we get the required new section modulus of 41.03 < 44.92 in^3. For the shear at x = 0.9708b is V = (32.03*0.2922825 + 468* 0.2919907) *40/2 = 2920.27 lbs we get the required area = 26.55 in^2 < 38.5.The strength is fine. If we look at the stress at x = 19.97 ft, y = 0 and is also close to y = 17 inch x 1 /p = 106.948 and x 2/q = 356.929. Or fb = (32.03*106.948+ 468*356.929)/(12*5.5) = 2582.85 psi > 2400 psi this is also on top of the elastic media so the fb of the beam without arching = 66,677.78*12/528 = 1515.24 psi. In here ν was taken as 0.3 so K_0 1 = 0.3 and K_0 2 = 1.0. If we look at K_0 2 = 0.3 for a rough surface contact, we have x 1 /p = 106.948 and x 2/q = 107.079. Or fb = (32.03*106.948 + 468*107.079)/(12*5.5) = 811.188 psi < 1515.24 psi and the beam appears stronger.

Even if we make the beam 10-inch high and the elastic media 14-inch high for maximum stress, A = 82944 the stress at x = 19.97, y = 0 and is also close to y = 17 inch x 1 /p = 111.087 and x 2/q = 370.808. Or fb = (32.03*111.087 + 468*370.808)/(12*5.5) = 2683 psi this is with K_0 2= 1.0. It is rare to have a smooth surface and K_0 2 is not 1.0. If we look at K_0 2 = 0.3 for a rough surface contact, we have x 1 /p = 111.087 and x 2/q = 111.242. Or fb = (32.03*111.087 + 468*111.242)/ (12*5.5) = 842.72 psi and the beam appear stronger.

Note: to calculate the reserve stress in a beam due to arching it is case by case because every beam is different.

Other solutions:

If solve for the coefficients *A*, *B*, *C* and *D* by setting σx at $x = t+b$ equal zero instead of setting the shear at $y = 0$ equal to zero and proceed with the derivation we can obtain the solution of Figure 4. If this is not possible than subtracting σx at $x = t+b$ is required. It is expected there will be more deflections and arching. Also, for unequaled span one can use the shorter span as an approximation to find the solution the domain of the Fourier series must be revised.

Future Work:

The solution for seismic moment will introduce a Fourier sine series in the deflection and the entire solution must be revised. It is left for future articles to cover other conditions such as a point load for live load and moments in the beam. Also, if add a beam at $y = h$ we can solve the problem for a wide flange beam where the flanges are the beams and the web is the mass media.

Conclusion:

It is evident from the charts that *h* is proportional to the arching reduction obtained. This article is worthy of or a candidate for a Ph. D. dissertation. The future research would involve systems' optimization with arching for any beam. The dividends of this research will produce great benefits for structural designs, not just for the Army Corps of Engineers, but for all Federal agencies, including the Forest Service, Aerospace Engineering (Air force), and Naval Architectures (Navy). The application in civil engineering includes culverts with ribs, tunneling with ribs, bridges with ribs, masonry buildings, concrete buildings, steel buildings and timber structures. The application in Aerospace and Naval Architectures is in reducing the weight of materials.

Reducing construction materials reduces energy consumption which reduces construction costs. In shoring and excavation systems, a 50% reduction from the empirical formula due to arching gives a 30% (√.5) reduction in materials (i.e. wood lagging over the entire face of the wall). This results in, approximately, a 15% reduction in total cost. Preliminary designs for deep culverts show 20% reduction in construction cost due to utilizing arching. If there is a reduction of 20% due to arching, as realized in the above charts for a moderate *h* for live or dead loads, then an average of 10% (√.8) reduction in materials can be achieved for most applications.

Can we say we caution most buildings in the world are safe because arching was never considered provided, they are designed with some code?

Acknowledgements:

Glory be to God. Thank you, Lord Jesus Christ, for giving me the mathematical solution in my meditation and help me afterward to understand the physics.

Appendix 1.-References

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